LOGIC (I)

- Logic: Its a knowledge representation technique where its output is either 0 or 1.

Logic (General Logic)

propositional   predicative

Propositional Logic :-

- In propositional logic we can represent a statement into a single proposition or literal.

ex: Ram is a boy = p
Hari is a boy = q → proposition

If Ram is a boy then Hari is a boy.

→ $p \rightarrow q$ → connectives

propositional statement

2-07-18

Syntax and Semantics of Propositional Logic

- In syntax it supports the following connectives to represent the propositional statements.

$\vee$ (disjunction)
$\wedge$ (conjuction)
$\sim$ (negation)
$\rightarrow$ (implication)
$\leftrightarrow$ (double implication)

Truth Table

(i) Disjunction : It will return a true value if either p or q is true.

- It will return a true value if both p & q are true.

(iii) Implication : It will return a false value if p is true & q is false

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

(iv) Double Implication : It will return a true value if both p & q are having same value

p	q	p↔q
T	T	T
T	F	F
F	T	F
F	F	T

Q: If Socretes is a man then he is mortal. Socretes is a man. Therefore Socretes is mortal.

ans:

Socretes is a man = p
Socretes is mortal = q

$p \to q$

p

$\therefore q$

$((p \to q) \land p) \to q \equiv$ Tautology

R

p	q	p→q	(p→q)∧p	R→q
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

- As it is found as tautology so the given statements are true.

Q: You can access the internet from campus only if you are a Computer Science major or you are not a freshman. Represent the sentences in propositional logic & check its validity.

ans:

Let p = you can access the internet
q : you are a computer science major
r : you are a fresh-man

$$p \rightarrow (q \vee \sim r)$$

p	q	r	$\sim r$	$q \vee \sim r$	$p \rightarrow (q \vee \sim r)$
T	T	T	F	T	T
T	T	F	T	~~F~~T	T
T	F	T	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	T	T	T
F	F	T	F	F	T
F	F	F	T	T	T

- As it is not found as tautology the given statements are in valid.

Q: If I look in to the sky & I am alert then either I will see the flying bird or if I am not alert and I will not see the flying bird.
Determine its truth table & check the validity of the satement.

ans:

Let p = I look in to the sky
q : I am alert
r = I see the flying bird

$(p \wedge q) \rightarrow r \vee (\sim q \wedge \sim r)$

M N

p	q	r	$p \wedge q$	$\sim q$	$\sim r$	$\sim q \wedge \sim r$	$r \vee (\sim q \wedge \sim r)$	$M \rightarrow N$
T	T	T	T	F	F	F	T	T
T	T	F	T	F	T	F	F	F
T	F	T	F	T	F	F	T	T
T	F	F	F	T	T	T	T	T
F	T	T	F	F	F	F	T	T
F	T	F	F	F	T	F	F	T
F	F	T	F	T	F	F	T	T
F	F	F	F	T	T	T	T	T

- As it is not found as tautology for the given statements are invalid.

Q: If baby is hungry then the babby cries.
If the baby is mad then he does not cry.
If a baby is mad then ~~the~~ he has a red face.
Therefore if a baby is hungry then he has a red face.

ans :

p : Baby is hungry
q : Baby ~~is mad~~ cries
r : Baby is mad
s : Baby has a red face.

$p \rightarrow q$
$r \rightarrow \sim q$
$r \rightarrow s$

$\therefore p \rightarrow s$

$((p \rightarrow q) \wedge (r \rightarrow \sim q) \wedge (r \rightarrow s)) \rightarrow (p \rightarrow s) \equiv$ tautology

				A			B	C	M	N	
p	q	r	s	$p \to q$	$\sim r$	$\sim q$	$\sim r \to \sim q$	$r \to s$	$A \wedge B \wedge C$	$p \to s$	$M \to N$
T	T	T	T	T	F	F	T	T	T	T	T
T	T	T	F	T	F	F	T	F	F	F	T
T	T	F	T	T	T	F	F	T	F	T	T
T	T	F	F	T	T	F	F	T	F	F	T
T	F	T	T	F	F	T	T	T	F	T	T
T	F	T	F	F	F	T	T	F	F	F	T
T	F	F	T	F	T	T	T	T	F	T	T
T	F	F	F	F	T	T	T	T	F	F	T
F	T	T	T	T	F	F	T	T	T	T	T
F	T	T	F	T	F	F	T	F	F	T	T
F	T	F	T	T	T	F	F	T	F	T	T
F	T	F	F	T	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T	T	T	T	T
F	F	T	F	T	F	T	T	F	F	T	T
F	F	F	T	T	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T	T	T	T

- As it is found as tautology for the given statements, the statements are valid.

Rules of Inference

- To deduce new statements from those whose truth values are already known is called inference rules.
- In propositional logic it supports the following inference rules.

(i) Modus Ponens:

- From P & ~~q~~ $P \to q$ we can infer q & logically it can be represented as

$$\begin{array}{c} P \\ \therefore\ P \to q \\ \hline q \end{array}$$

$$(P \land (P \to q)) \to q \equiv \text{tautology}$$

P	q	$P \to q$	$\overset{R}{\overline{P \land (P \to q)}}$	$R \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Valid

(ii) Rule of Conjection:

- From P & q we can infer $P \land q$ & logically it can be represented as

$$\begin{array}{c} P \\ q \\ \hline \therefore\ P \land q \end{array}$$

$$(P \land q) \to P \land q \equiv \text{tautology}$$

(iii) Rule of disjunction:

- From P & q we can infer $P \lor q$
- logically can be represented as

$$\begin{array}{c} P \\ q \\ \hline \therefore\ P \lor q \end{array}$$

$(p \wedge q) \rightarrow (p \vee q) \equiv$ tautology

$\underbrace{\quad}_{}$ R $\quad$ S

p	q	$p \wedge q$	$p \vee q$	$R \rightarrow S$	
T	T	T	T	T	
T	F	F	T	T	Valid
F	T	F	F	T	
F	F	F	T	T	

(iv) Rule of implication :

- From P & q we can infer $p \rightarrow q$
- mathematically can be represented as

$$\begin{array}{l} p \\ \underline{: q} \\ \therefore p \rightarrow q \end{array}$$

$(p \wedge q) \rightarrow (p \rightarrow q) \equiv$ tautology

p	q	$p \wedge q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$	
T	T	T	T	T	
T	F	F	F	T	Valid
F	T	F	T	T	
F	F	F	T	T	

(v) Chain Rule :

- From $p \rightarrow q$ & $q \rightarrow r$ we can infer $p \rightarrow r$

mathematically it can be written as

$$\begin{array}{l} p \rightarrow q \\ \underline{q \rightarrow r} \\ \therefore p \rightarrow r \end{array}$$

$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r) \equiv$ tautology

1			A	B	C	M		
p	q	r	$p \to q$	$q \to r$	$p \to r$	$A \land B$	$M \to C$	
T	T	T	T	T	T	T	T	
T	T	F	T	F	F	F	T	
T	F	T	F	T	T	F	T	
T	F	F	F	T	F	F	T	valid
F	T	T	T	T	T	T	T	
F	T	F	T	F	T	F	T	
F	F	T	T	T	T	T	T	
F	F	F	T	T	T	T	T	

Normalisation:

- It is a technique to simplyfy propositional statements.
- In general there are two types of normal forms
  i.e CNF (Conjuctive Normal Form)
  DNF (Disjuctive Normal Form)

CNF :-

- It can be defined as conjuction of disjunction of literals. i.e

$(A \lor B) \land (A \lor C) \land (B \lor C) \land \ldots$

similar as (POS)

DNF :-

- It can be defined as disjunction of conjuction of literals. i.e

$(A \land B) \lor (A \land C) \lor (B \land C) \lor \ldots$

similar as (SOP)

Procedure to Convert in to Normal Forms

Stp-1 : Eleminate all impleecation (→) & doubleimplication(↔) symbols by using the following formulae.

$$p \to q \equiv \sim p \vee q$$
$$p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$$

Stp-2: Eleminate all negation symbols that are in grouping manner by using the following formulas.

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$
$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$
$$\sim(\sim p) \equiv p$$

Stp-3: Convert the statement in to CNF or DNF by using the following rules.

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$
$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \wedge r)$$

Stp-4 : END

Q: Convert the following statements into DNF

1> $p \to (q \to r)$

$\Leftrightarrow$ $p \to (\sim q \vee r)$

$\Leftrightarrow$ $\sim p \vee (\sim q \vee r)$

$\Leftrightarrow$ $\sim p \vee \sim q \vee r$

Q: Convert in to CNF

2> $(p \to q) - r$

$\Leftrightarrow$ $(\sim p \vee q) \to r$

$\Leftrightarrow$ $\sim(\sim p \vee q) \vee r$

$\Leftrightarrow$ $(\sim(\sim p) \overset{\wedge}{\bullet} \sim q) \vee r$

$\Leftrightarrow$ $(p \wedge \sim q) \vee r$

$\Leftrightarrow$ $(p \vee r) \wedge (\sim q \vee r)$

Convert to DNF

$(p \vee r) \wedge ((q \vee p) \wedge r$

$\Leftrightarrow (p \vee r) \wedge ((q \wedge r) \vee (p \wedge r))$

$\Leftrightarrow ((p \vee r) \wedge (q \vee r)) \vee ((p \vee r) \wedge (p \wedge r))$

$\Leftrightarrow ((p \wedge q \wedge r) \vee (r \wedge q \wedge r)) \vee ((p \wedge q \wedge r) \vee (r \wedge p \wedge r))$

$\Leftrightarrow (p \wedge q \wedge r) \vee (r \wedge q \wedge r) \vee (p \wedge p \wedge r) \vee (r \wedge p \wedge r)$

Convert do DNF

$\sim(p \wedge \sim q) \rightarrow (r \vee s)$

$\Leftrightarrow (\sim p \vee \sim(\sim q)) \rightarrow (r \vee s)$

$\Leftrightarrow (\sim p \vee q) \rightarrow (r \vee s)$

$\Leftrightarrow \sim(\sim p \vee q) \vee (r \vee s)$

$\Leftrightarrow (\sim(\sim p) \wedge \sim q) \vee (r \vee s)$

$\Leftrightarrow (p \wedge \sim q) \vee (r \vee s)$

$\Leftrightarrow ((p \wedge \sim q) \vee r) \vee ((p \wedge \sim q) \vee s)$

$\Leftrightarrow (p \wedge \sim q) \vee r \vee (p \wedge \sim q) \vee s$

Predicate Logic (FOPL - First Order Predicate Logic)

(or) WFF. Well Formed Formulae

eg: Ram is eating an apple. → Peredicate
S V O

→ Constant ←

EATING (ram, apple)

E(ram, apple)

where E = eating

Ram is a doctor.

D(ram) where D: Doctor

All Ram are eating apple.

$\forall x$: ~~Ram~~ $RAM(x) \rightarrow E(x, apple)$

where E = eating

{All, every} $\forall$
{any, some, atleast} $\exists$

All Ram are eating some apple.

$(\forall x)(\exists y) : R(x) \wedge A(y) \rightarrow E(x, y)$

where R : Ram
A : Apple
E : Eating

Everybody likes Somebody.

$(\forall x)(\exists y) : L(x, y)$

Every Ram likes some Hari.

$(\forall x)(\exists y) : R(x) \wedge H(y) \rightarrow L(x, y)$

There is no prime number in between 23 & 27.

$(\exists x) : P(x) \rightarrow \sim Pr(x, 23, 27)$

$(\exists x) : P(x) \rightarrow \sim Pr(x, 23, 27)$

Ram is neither a professor nor a doctor.
→ $\sim(D(ram) \wedge P(ram))$
→ $(\exists x): R(x) \rightarrow \sim(D(x) \wedge P(x))$

Somebody is liked by everybody.
$(\exists x)(\forall y): L(x,y)$ where L: liked by

There is rooten in the state of Danemark.
$(\exists x): R(x) \rightarrow P(x, denmark)$
→ $(\exists x): R(x) \wedge P(x, denmark)$
where R = rooten, P = present in the state of

Syntax and Semantics of FOPL

{FOPL = First Order Predicate Logic}

Syntax:

- In syntax it supports the following components to represent predicate logic.

- Connectives: It includes five symbols for represention as
  - → implication
  - ↔ double implication
  - ~ negation
  - ∧ conjuction
  - ∨ disjunction

- Quantifiers: It includes two quantifiers
  - ∀ universal quantifier
  - ∃ existantial quantifier

- Variables: It includes some English alphabets in terms of lowercase letters such as x, y, z --

Constants: It includes some English alphabets or a single alphabet in terms of lower case letters

- Constants are generally treated as fixed values

eg: Ram, apple, a, 33....

Predicates: In includes a single alphabet or a group of alphabets in terms of uppercase letters.

eg. R, H, ...

Functions: It indicates the represantation for a specific domain.

eg: f(x), g(a), h(g)

Semantics:

Rule-1: Subjects & objects of the sentence must be used inside the paranthesis.

Rule-2: Predicate of the sentence must be used outside the paranthesis.

Rule-3: Instead of the complex words like every, all etc we can use universal quantifier.

Rule 4: Instead of ~~of~~ the complex works like any, some, atleast etc. we can use existantial quantifier.

Claeser Algorithm:

- It is the simplest form of FOPL statements i.e it should not contain symbols like →, ↔, ∧, ∨ in groupism.

Stp-1

Eliminate all implications and double implecation symbols by using the Following Formula

$$P(x) \rightarrow Q(y) \equiv \sim P(x) \lor Q(y).$$
$$P(x) \leftrightarrow Q(y) \equiv (\sim P(x) \lor Q(y)) \land (\sim Q(y) \lor P(x))$$

Stp-2

- Eliminate all negation symbols that are in grouping manner by using the Following Formula -

$$\sim(P(x) \land Q(x)) \equiv \sim P(x) \lor \sim Q(x)$$
$$\sim(P(x) \lor Q(x)) \equiv \sim P(x) \land \sim Q(x)$$
$$\sim(\sim P(x)) \equiv P(x)$$

Stp-3

- Convert the statement into CNF Form.

Stp-4

- Retrive each term at each conjuction point & call them as clause.

Stp-5

Exit

eg: Find out the no. of clauses of the statement
$$(P(x) \rightarrow Q(x)) \rightarrow R(x)$$

$$\Leftrightarrow (\sim P(x) \lor Q(x)) \rightarrow R(x)$$

$$\Leftrightarrow \sim(\sim P(x) \lor Q(x)) \lor R(x)$$

$$\Leftrightarrow (\sim(\sim P(x)) \land \sim Q(x)) \lor R(x).$$

$$\Leftrightarrow (P(x) \land \sim Q(x)) \lor R(x)$$

$\Leftrightarrow \left(P(x) \vee R(x)\right) \wedge \left(\sim Q(x) \vee R(x)\right)$

$P(x) \vee R(x)$ —— ①

$\sim Q(x) \vee R(x)$ —— ②

Resollution :

- It is a technique to check the validity of logical statements.

Stp-1

- Convert all the FOPL statements in to their clause form

Stp-2

- Find out the negation of the goal statement, convert it in to clause form, and add it with the set of clauses found in step 1.

Stp-3

- Select any two clauses and call them as parent clauses such that "L" is a literal of one clause and "~L" is a literal of another clause.

Stp-4

- Resolve the parent clauses togather by the method of substitution, & it will produce a new clause called "resolvant" i.e the disjunction of all literals of the parent clauses.

Stp-5

- If the resolvant is found as emptey then contradictio will occur, so stop & return success, otherwise go to stp-3 for finding out another parent clause and repeat the process.

Stp-6 . Exit

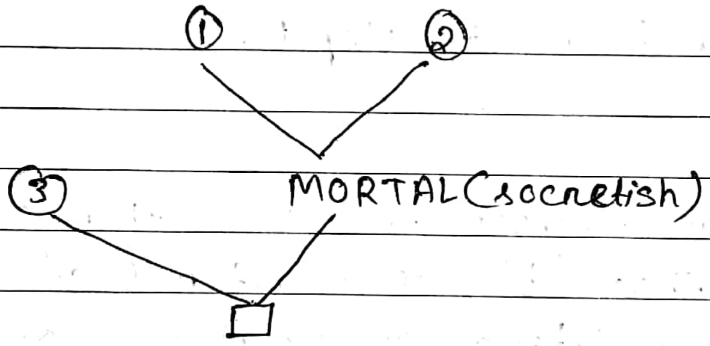
eg: If Socretish is a man then he is mortal.
Socretish is a man. Therefore he is mortal.

ans:

Socretish is man : MAN(socretish)
Socretish is mortal: MORTAL(socretish

$$\frac{\begin{array}{c}MAN(socretish) \rightarrow MORTAL(socretish)\\ MAN(socretish)\end{array}}{\therefore MORTAL(socretish)}$$

$\sim MAN(socretish) \vee MORTAL(socretish)$ —— ①
$MAN(socretish)$ —— ②
$\sim MORTAL(socretish)$ —— ③



∴ As the contradiction is found the given statements are valid.

Q. Check the validity of given statement using resol^n.

If a baby is hungry then the baby cries.
If the baby is not mad then the baby does not cry.
If a baby is mad then he has a red face.
Therefore if a baby is hungry then he has a red face.

Baby is hungry : H(baby)
Baby cries : C(baby)
Baby is mad : M(baby)
Baby has a redface : R(baby)

$H(baby) \rightarrow C(baby)$
$\sim M(baby) \rightarrow \sim C(baby)$
$\underline{M(baby) \rightarrow R(baby)}$
$\therefore H(baby) \rightarrow R(baby)$

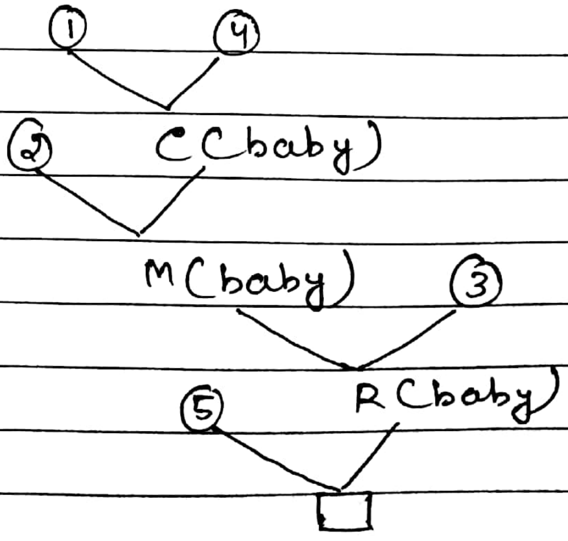
$\sim H(baby) \vee C(baby)$ ——— ①

$\sim(\sim M(baby)) \vee \sim C(baby)$
$\Leftrightarrow M(baby) \vee \sim C(baby)$ ——— ②

$\sim M(baby) \vee R(baby)$ ——— ③

$\sim(H(baby) \rightarrow R(baby))$
$\Leftrightarrow \sim(\sim H(baby) \vee R(baby))$
$\Leftrightarrow \sim(\sim H(baby)) \wedge \sim R(baby)$
$\Leftrightarrow H(baby) \wedge \sim R(baby)$
$\Leftrightarrow H(baby)$ ——— ④
$\sim R(baby)$ ——— ⑤

- As the contradiction is found, so the given statements are valid



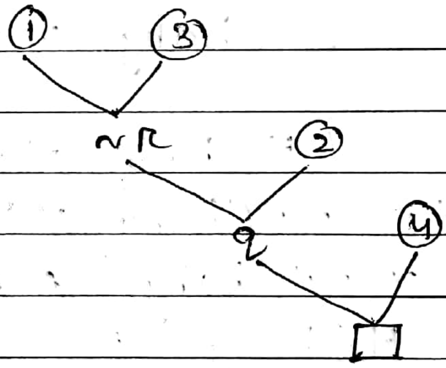
Q. Using resolution show that the hypothesis is valid.

"Jasmine is skiing or it is not snowing" & "It is snowing or John is playing hockey" implies that "Jasmine is skiing or John is playing hockey"

soln: Jasmine is skiing = p
John is playing hockey = q
It is snowing = r

$p \vee \sim r$
$\underline{r \vee q}$
$\therefore p \vee q$

$p \vee \sim r$ ——— ①
$r \vee q$ ——— ②
$\sim(p \vee q)$
$\Leftrightarrow \sim p \wedge \sim q$
$\sim p$ ——— ③
$\sim q$ ——— ④



- As contradiction is found the statements are valid.

Logical Equivalance Table:

Equivalance	Name	Equivalance	Name
$p \wedge T \equiv p$ $p \vee F \equiv p$	Identity Law	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive Law
$p \vee T \equiv T$ $p \wedge F \equiv F$	Domination Law	$\sim (p \vee q) \equiv \sim p \wedge \sim q$ $\sim (p \wedge q) \equiv \sim p \vee \sim q$	Demorgan's Law
$p \vee p \equiv p$ $p \wedge p \equiv p$	Ideopotent Law	$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption Law
$\sim (\sim p) \equiv p$	Double negation Low	$p \vee \sim p \equiv T$ $p \wedge \sim p \equiv F$	Negation Law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative Laws	$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative Law

Logical Equivalance:

- The compared positions A and B are called Logical equivalance. if "A → B" is a "tautology" then notation "A = B" denotes that 'A' & 'B' are Logically equivalent i.e they have the truth value.

Q. Show that $p \to q$ & $\sim p \vee q$ are logically equivalent. using truth table.

p	q	$p \to q$	$\sim p$	$\sim p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

→ logical equivalance

Not using truth table:

$p \to q \equiv \sim p \vee q$ [∵ Laws of implication]

Q. Show that $\sim(p \vee (\sim p \wedge q))$ & $\sim p \wedge \sim q$ are logically equivalance

$\sim(p \vee (\sim p \wedge q)) \equiv (\sim p \wedge \sim(\sim p \wedge q))$ {Demorgans Law}

$\equiv (\sim p \wedge (\sim(\sim p) \vee \sim q))$ {Demorgan's Law}

$\equiv (\sim p \wedge (p \vee \sim q))$ {Double Negation Law}

$\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$ {Distributive Law}

$\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$ {Distributive Law}

$\equiv F \vee (\sim p \wedge \sim q)$ {Negation Law}

$\equiv (\sim p \wedge \sim q)$ {Identity Law}

Q. Show that $p \wedge q \to p \vee q$ is a tautology without showing thruth table

$p \wedge q \to p \vee q$

$\equiv \sim(p \wedge q) \vee (p \vee q)$ [∵ Implication Law]

$\equiv (\sim p \vee \sim q) \vee (p \vee q)$ [∵ Demorgan's Law]

$\equiv (\sim p \vee p) \vee (\sim q \vee q)$ [∵ Associative Law]

$\equiv T \vee T$ [∵ Negation Law]

$\equiv T$ [∵ Domination Law]

Q. Show that $\sim(P \to q)$ & $P \land \sim q$ are logically equivalent

$\sim(P \to q)$

$\equiv \sim(\sim P \lor q)$ [∵ Implication Law]

$\equiv \sim(\sim P) \land \sim q$ [∵ Demorgan's Law]

$\equiv P \land \sim q$ [∵ Double negation]

Mathematical Induction :

- It is a mathematical technique which is used to prove a statement, formula or theorem for every natural no.
- It has two parts
  - (i) Basic Steps
  - (ii) Inductive Steps

- In Basic steps we have to show wheather the statement is true for n = 0 (non-negative nos) & n = 1 (positive nos)
- In Inductive steps we have to assume 1st the statement is true at n = k and then we have to proof the statement will be true at n = k+1
- If the statement is found as true at n = k+1 then it is ~~ed~~ ~~on~~ concluded that the statement is true for n = m by the method of induction.

Q. Show that if n is a +ve integer than

$$1+2+3+\cdots\cdots+n = \frac{n(n+1)}{2}$$ by the method of induction

Soln : Let $P(n) = 1+2+\cdots+n = \frac{n(n+1)}{2}$

Stp-1 at $n = 1$

LHS : $P(1) = 1$

RHS : $\frac{n(n+1)}{2} = \frac{1(1+1)}{2} = 1$

LHS = RHS

Stp-3

Let us assume the statement true for n = k

$\Rightarrow P(k) = 1+2+3+ \dots +k = \frac{k(k+1)}{2}$ — ①

Stp-3 at $n = k+1$

$P(k+1) = 1+2+3+ \dots + k + k+1 = \frac{(k+1)(k+2)}{2}$

LHS.

$1+2+3+ \dots + k + k+1$

$= \frac{k(k+1)}{2} + k+1$

$= \frac{k(k+1)+2(k+1)}{2}$

$= \frac{(k+1)(k+2)}{2}$ = RHS

LHS = RHS

Now it is concluded that the statement is true of n is a +ve integer.

Q. $1+2+2^2+ \dots +2^n = 2^{n+1} -1$

n = non-negative integer

Soln:

Let $P(n) = 1+2^1+2^2+ \dots +2^n = 2^{n+1} -1$

Stp-1 at $n=0$

LHS $p(0) = 1$

RHS $p(0)$

$= 2^{0+1} -1 = 2-1 = 1$

LHS = RHS

Stp-2 Let us assume that the statement is true at $n = k$

$\Rightarrow P(k) = 1+2+2^2+ \dots +2^k = 2^{k+1} -1$ — ①

Stp-3 at $n = k+1$

$$\Rightarrow P(k+1) : 1+2+2^2+\dots+2^k+2^{k+1} = 2^{k+2}-1 \quad ②$$

LHS =

$$1+2+2^2+\dots\dots+2^k+2^{k+1}$$

$$= 2^{k+1}-1+2^{k+1}$$

$$= 2\cdot 2^{k+1}-1$$

$$= 2^1\cdot 2^{k+1}-1 = 2^{k+2}-1 = \text{RHS}$$

Q. Show that

$$1^2+2^2+3^2+\dots\dots+n^2 = \frac{n(n+1)(2n+1)}{6}, \quad \forall\, n \geq 1$$

Soln:

Let $P(n) : 1^2+2^2+3^2+\dots\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$

Stp-1 $n = 1$

LHS. $P(1) = 1$

RHS. $\frac{1(1+1)(2+1)}{6} = \frac{2\times 3}{6} = \frac{6}{6} = 1$

LHS = RHS

Stp-2

Let us assume the statement is true at $n = k$

$$P(k) = 1^2+2^2+3^2+\dots\dots+k^2 = \frac{k(k+1)(2k+1)}{6}$$

Stp-3

at $n = k+1$

$$P(k+1) = 1^2+2^2+\dots\dots+k^2+(k+1)^2 = \frac{(k+1)(k+2)(2(k+1)+1)}{6}$$

$$P(k+1) = 1^2+2^2+\dots\dots+k^2+(k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$

LHS.

$1^2+2^2+3^2+\dots+k^2+(k+1)^2$

$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$

$= \frac{k(k+1)(2k+1)+6(k+1)^2}{6}$

$= \frac{(k+1)(k(2k+1)+6(k+1))}{6}$

$= \frac{(k+1)(2k^2+k+6k+6)}{6}$

$= \frac{(k+1)(2k^2+7k+6)}{6}$ $= \frac{(k+1)(2k^2+2k+3k+6)}{6}$

$= \frac{(k+1)(2k(k+2)+3(k+2))}{6}$

$= \frac{(k+1)(k+2)(2k+3)}{6} = \text{RHS}$

Q. Show that n^3-n is divisible by 3 for all +ve integers.

soln: let $P(n): n^3-n$

Stp-1

$P(1) = 1-1 = 0$ is divisible by 3

Stp-2

let assume the statement is true for $n=k$

$P(k): k^3-k$ is divisible by 3

Stp-3

one task is to show the statement will be true at $n=k+1$

i.e $P(k+1) = (k+1)^3 - (k+1)$

$= k^3+1+3k^2+3k-k-1$

$= k^3-k+3k^2+3k$

$= 3m+3(k^2+k) = 3(m+k^2+k)$

So the expression n^3-n is divisible by 3, $\forall\ n \geq 1$

~~Imp~~ Q. Use mathematical induction, show that $2^n < n!$ for every integer n, with $n \geq 4$

Soln: $2^n < n!$

Let $P(n) = 2^n < n!$

Step-1 at $n = 4$

$2^4 < 4!$

$\Leftrightarrow 16 < 24$ (true)

Step-2

Let us assume the statement is true at

$n = k \Rightarrow P(k) = 2^k < k!$ —— ①

Step.3 at $n = k+1$

$\Rightarrow P(k+1) = 2^{k+1} < (k+1)!$

LHS. $2^{k+1} = 2^k \cdot 2$

$\leq (k+1) \cdot 2^k$ [∵ $2 \leq (k+1)$]

$< (k+1)\,k!$ [∵ at equn ①]

$< (k+1)! = RHS.$

Q. Use mathematical induction to prove the formula for the sum of a finite no. of terms of geometric progression.

$$a + ar + ar^2 + \cdots + ar^n = \frac{ar^{n+1} - a}{r-1},\quad r \neq 1$$

Proof:

Let $P(n) = a + ar + ar^2 + \dots + ar^n = \frac{ar^{n+1} - a}{r-1}, r \neq 1$

Stp-1

for $n = 0$

LHS. $P(0) = ar^0 = a$

RHS : $\frac{ar^{0+1} - a}{r-1} = \frac{ar-a}{r-1} = \frac{a(r-1)}{r-1} = a$

LHS = RHS

Stp-2

Let us assume the statement is true for $P(k)$

$\Rightarrow P(k) = a + ar + ar^2 + \dots + ar^k = \frac{ar^{k+1}}{r-1}$ ——— ①

Stp-3 at $n = k+1$

$P(k+1) = a + ar + ar^2 + \dots + ar^k + ar^{k+1} = \frac{ar^{k+2} - a}{r-1}$ — ②

LHS. $\underline{a + ar + ar^2 + \dots + ar^k} + ar^{k+1}$

$$= \frac{ar^{k+1} - a}{r-1} + ar^{k+1}$$

$$= \frac{ar^{k+1} - a + ar^{k+1}(r-1)}{r-1}$$

$$= \frac{ar^{k+1} - a + ar^{k+2} - ar^{k+1}}{r+1}$$

$$= \frac{ar^{k+2} - a}{r+1} = \text{RHS}$$

∴ The statement is true for $n \neq 1$ & n is a non-negative integer.

Q. $1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)$ where n is +ve integer

Stp-1 Let $P(n) = 1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

at $n=1$

LHS. $P(1) = 1^3 = 1$

RHS $= \left(\frac{1(1+1)}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = (1)^2 = 1$

LHS = RHS

Stp-2

Let us assume $P(n)$ is true for $n = k$

$\Rightarrow P(k) = 1^3 + 2^3 + 3^3 + \dots + k^3 = \left(\frac{k(k+1)}{2}\right)^2$ —— ①

Stp-3

at $n = k+1$

$\Rightarrow P(k+1) = 1^3 + 2^3 + 3^3 + \dots + k^3 + (k+1)^3 = \left(\frac{(k+1)(k+2)}{2}\right)^2$ ②

LHS $= 1^3 + 2^3 + 3^3 + \dots + k^3 + (k+1)^3$

$= \left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3$ {∵ equn ①}

$= (k+1)\left[\frac{k^2}{4} + (k+1)\right]$

$= (k+1)^2\left[\frac{k^2+4k+4}{4}\right]$

$= (k+1)^2\left[\frac{(k+2)^2}{2^2}\right]$

$= \left(\frac{(k+1)(k+2)}{2}\right)^2 =$ RHS

The statement is true for n is a +ve integer

Q. Prove that $\sum_{j=0}^{n} \left(-\frac{1}{2}\right)^j = \frac{2^{n+1} + (-1)^n}{3 \cdot 2^n}$

where n is a non-negative integer

Soln: Let $P(n) = \sum_{j=0}^{n} \left(-\frac{1}{2}\right)^j = \frac{2^{n+1} + (-1)^n}{3 \cdot 2^n}$

Step-1 at $n=0$

$P(0) = \left(-\frac{1}{2}\right)^0 = 1 = LHS$

$RHS = \frac{2^{0+1} + (-1)^0}{3 \cdot 2^0} = \frac{2+1}{3} = \frac{3}{3} = 1 =$

LHS = RHS

Step-2

Let us assume P(n) is true for n = k

i.e $P(k) = \sum_{j=0}^{k} \left(-\frac{1}{2}\right)^j = \frac{2^{k+1} + (-1)^k}{3 \cdot 2^k}$ —— ①

Step-3

at $n = k+1$

$P(k+1) = \sum_{j=0}^{k+1} 3 \cdot 5^j = \frac{3(5^{k+2} - 1)}{4}$

$LHS = \sum_{j=0}^{k} 3 \cdot 5^j + 3 \cdot 5^{k+1}$

$= \frac{3(5^{k+1} - 1)}{4} + 3 \cdot 5^{k+1}$

$= \frac{3 \cdot 5^{k+1} - 3 + 4 \cdot 3 \cdot 5^{k+1}}{4}$

$= \frac{3(5^{k+1} + 4 \cdot 5^{k+1} - 1)}{4} = \frac{3(5 \cdot 5^{k+1} - 1)}{4} = \frac{3(5^{k+2} - 1)}{4}$

$= RHS$

LHS = RHS

it is proved that $3 + 3 \cdot 5 + 3 \cdot 5^2 + \cdots + 3 \cdot 5^n = \frac{3(5^{n+1} - 1)}{4}$

~~for~~ where n is a non-negative integer.

Q. $1^2+3^2+5^2+\dots\dots+(2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$

n is non-negative integer

Proof: Let $P(n) = 1^2+3^2+5^2+\dots\dots+(2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$

Step-1 at $n=0$

$LHS = P(0) = 1^2 = 1$

$RHS = \frac{1.3}{3} = \frac{3}{3} = 1$

$LHS = RHS$

Step-2

Let us assume the $P(n)$ is true for $n=k$

$P(k) = 1^2+3^2+5^2+\dots\dots+(2k+1)^2 = \frac{(k+1)(2k+1)(2k+3)}{3}$ — ①

Step-3 at $n=k+1$

$P(k+1) = 1^2+3^2+5^2+\dots\dots+(2k+1)^2+(2(k+1)+1)^2 = \frac{\{(k+1)+1\}\{2(k+1)+1\}\{2(k+1)+3\}}{3}$ — ②

LHS =

$1^2+3^2+5^2+\dots+(2k+1)^2+\{2(k+1)+1\}^2$

$= \frac{(k+1)(2k+1)(2k+3)}{3} + (2(k+1)+1)^2$

$= \frac{(k+1)(2k+1)(2k+3)+3(2k+3)^2}{3}$

$= \frac{(2k+3)[(k+1)(2k+1)+3(2k+3)]}{3}$

$= \frac{(2k+3)}{3}[(2k^2+k+2k+1)+3(2k+3)]$

$= \frac{(2k+3)}{3}[2k^2+3k+1+6k+9]$

$= \frac{(2k+3)}{3}(2k^2+9k+10)$

$= \frac{(2k+3)}{3}(2k^2+4k+5k+10)$

$$= \frac{(2k+3)\left[2k(k+2)+5(k+2)\right]}{3}$$

$$= \frac{(2k+3)(2k+5)(k+2)}{3}$$

$$= \frac{(2(k+1)+1)(2(k+1)+3)((k+1)+1)}{3}$$

$$= \frac{\left[(k+1)+1\right]\left[2(k+1)+1\right]\left[2(k+1)+3\right]}{3}$$

= RHS

∴ The expression is true for n is a ~~non-magnitude~~ non-negative integer.

Introduction to Proof :-

Q. If x^2 is odd then so x^2 is odd

soln: Let us consider x is even, then according to definition of even no.s we can ~~[illegible]~~ define

$x = 2n$

$\Rightarrow x^2 = 4n^2$

$= 2 \cdot 2n^2$

$=$ even

So the above process contradicts that x^2 is odd

Q. Proof that if n is an integer and $3n+2$ is odd then n is odd.

proof:

Let us consider n is even then we can write

$n = 2k$ [$\because$ Definition of even]

So $3n+2 = 3 \cdot 2k + 2$

$= 6k+2$

$= 2(3k+1)$

$=$ even

So the above process contradicts that n is odd

Q. If x^2+x is even prove that x is even

proof: Let us consider x is even

case-1 then $x = 2k$ [$\because$ deffinition of even no]

$\Rightarrow x^2 + x = (2k)^2 + 2k$

$= 4k^2 + 2k$

$= 2(2k^2+k) =$ even proved

case-2 Let us consider x is odd then x can be written as $x = 2k+1$ [$\because$ deffinition of odd]

$\Rightarrow x^2+x = (2k+1)^2 + (2k+1)$

$= 4k^2+1+4k+2k+1$

$= 4k^2+6k+2$

$= 2(2k^2+3k+1) =$ even proof.

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Methods of Proof

- A proof is a valid argument that establishes the truth of a theorem.
- There are different methods of proving such as

(i) Direct Proof

- A direct Proof is a ~~valid~~ argument ~~that~~ conditional
$p \to q$ statement
- Here 1st we will assume that P is true and by applying different axioms, definitions, rules of ~~interferen~~ inferences, we have to show q is also true

eg: Show that if n is odd then n^2 is also odd

n is odd $\Rightarrow n^2$ is odd

proof: Let n is odd

$\Rightarrow n = 2k+1$, where k is an integer

$\Rightarrow n^2 = (2k+1)^2$

$= n^2 = 4k^2+1+4k$

$= 2(2k^2+2k)+1$

$= 2m+1$ (where $m = 2k^2+2k$)

$=$ Odd (proof).

(ii) Proof by Contraposition

- The proof by contraposition makes the use of the equivalance.

$$p \to q \equiv \sim q \to \sim P$$

here 1st we assume $\sim q$ is true & by apply definitions, axioms, rules of inferences to show $\sim P$ is also true

eg: show that if $3n+2$ is an odd integer then n is odd

proof: Let us assume n is even

$\Rightarrow n = 2k$ where k is an integer

$\Rightarrow 3n+2 = 3(2k)+2$

$\Rightarrow 3n+2 = 6k+2$

$= 2(3k+1)$

$= 2m$ {where $m = 3k+1$}

= even proof.

(iii) Proof by Contradiction

- This method makes use of the equivalance

eg. $p \equiv \sim p \to F_0$

where F_0 is any contradiction

here 1st assume $\sim p$ is true and then show that For some proposition r,

r is true and $\sim r$ is also true

eg. show that if $3n+2$ is an odd integer then n is odd

proof: let us assume n is even

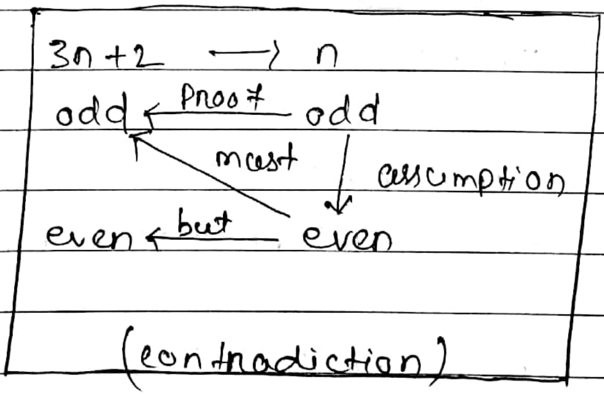
$\Rightarrow n = 2k$ (where k is an integer)

$\Rightarrow 3n+2 = 3(2k+2)$

$= 6k+2$

$= 2(3k+2)$

$= 2m$ (where $m = 3k+2$)



(iv) Proof by Cases

- This type of method of proooving can be applied by viewing on different cases

eg. show that if an integer n is not divisible by 3 then $n^2 = 3k+1$ for some integer k.

Proof

It is given that n is not divisible by 3

i.e equivalent to $n = 3m+1$ (or) $n = 3m+2$ (where m is an integer)

case-1

Let $n = 3m+1$

$$\Rightarrow n^2 = (3m+1)^2$$
$$= 9m^2 + 1 + 6m$$
$$= 3(3m^2 + 2m) + 1$$
$$= 3k+1 \text{ (where } k = 3m^2 + 2m)$$

case-2

$n = 3m+2$

$$\Rightarrow n^2 = (3n+2)^2$$
$$= 9m^2 + 4 + 12m$$
$$= 9m^2 + 12m + 4$$
$$= 9m^2 + 12m + 3 + 1$$
$$= 3(3m^2 + 4m + 1) + 1$$
$$= 3k+1 \text{ (where } k = 3m^2 + 4m + 1)$$

on viewing the above cases it is concluded that the original statement is true.

(v) Proof by Exhautive Method

- Some theorems can be proved by examining relatively small no. of examples and such proves are called exhaustive proves.

eg: prove that $(n+1)^3 \geq 3^n$ if n is a +ve integer with $n \leq 4$

case-1

Let $n = 1$

$\Rightarrow (1+1)^2 = 8 \geq 3^1$

case-2

Let $n = 2$

$\Rightarrow (2+1)^3 = 27 \geq 3^2$

case-3

Let $n = 3$

$\Rightarrow (3+1)^3 = 64 \geq 3^3$

case-4

Let $n = 4$

$\Rightarrow (4+1)^3 = 125 \geq 3^4$

In each of these 4 cases we see that $(n+1)^3 \geq 3^n$ and by using the method of exhaustion to prove that $(n+1)^3 \geq 3^n$

Q. Use direct prove to show sum of two odd integers is even.

proof: Let us assume m & n are two odd integers

$\Rightarrow m = 2k+1, \ n = 2l+1$ [where k, l are two integers]

$m+n = 2k+1+2l+1$

$= 2k+2l+2$

$= 2(k+l+1) = 2P$ where $P = k+l+1$

$=$ even

Q. Use direct prove to show the sum of two even integer is even

Proof: Let us assume m & n are two even integers

$\Rightarrow m = 2k, n = 2l$

$m+n = 2k + 2l$

$= 2(k+l)$

$= 2P$ (where $P = k+l$)

$=$ even (proved)

Q. Show that if n is an integer and n^3+5 is odd then n is even using

(i) a proof by contraposition

(ii) a proof by contradiction

soln) given n^3+5 is odd $\rightarrow$ n is even

(i) Contraposition

$P \rightarrow q \equiv \sim P \rightarrow \sim p$

Let us assume n is odd and we have to show n^3+5 is even

$\Rightarrow n = 2k+1$ [where k is an integer]

$\Rightarrow n^3+5 = (2k+1)^3 + 5$

$= 8k^3 + 6k + 12k^2 + 1 + 5$

$= 8k^3 + 6k + 12k^2 + 6$

$= 2(4k^3 + 3k + 6k^2 + 3)$

$= 2P$ [where $P = 4k^3 + 3k + 6k^2 + 3$

$=$ even $=$ integer]

(ii) Contradiction $P \wedge \sim P$ is true

Let us assume n is odd and we have to show n^3+5 is also odd

$\Rightarrow n = 2k+1$ (where k is an integer)

$\Rightarrow n^3+5 = 8k^3 + 1 + 6k + 12k^2 + 5$

$= 8k^3 + 6k + 12k^2 + 6$

$= 2(4k^3 + 3k + 6k^3 + 3)$

$= 2P$ where $P = 4k^3 + 3k + 6k^3 + 3$

$=$ even

which occurs, contradiction that indicates n^3+5 & n are both odd is wrong.

Q. Prove that n is +ve integer than n is even if and only if (iff) $7n+4$ is even

proof: Let us consider n is a +ve even integer

then $p \to q \Leftrightarrow \sim q \to \sim p$

$7n+4$ is even $\to$ n is even

then n is even

$\Rightarrow n = 2k$ (where k is any integer)

$\Rightarrow 7n+4 = 7(2k)+4$

$= 14k + 4$

$= 2(7k+2)$

$= 2P$ {where $P = 7k+2$}

even.

To prove the converse we use the method of contraposition i.e let us assume n is odd & we have to show $7n+4$ is odd

i.e $n = 2k+1$

$\Rightarrow 7n+4 = 7(2k+1)+4$

$= 14k+7+4$

$= 14k+11$

$= 14k+10+1$

$= 2(7k+5)+1$

$= 2P+1$ (where $P = 7k+5$)

$=$ odd

// Q. Prove that if n is +ve integer then n is odd iff $5n+6$ is odd

Proof. Let us consider n is a +ve odd integer
$5n+6$ is odd $\leftrightarrow$ n is odd

Let us assume n is odd, we have to show $5n+6$ is odd
$\Rightarrow n = 2k+1$ (where k is an ~~int~~ integer)
$\Rightarrow 5n+6 = 5(2k+1)+6 = 10k+5+6 = 10k+11 = 10k+10+1$
$= 2(5k+5)+1$
$= 2p+1$
(let $5k+5 = p$)
= odd

To prove the converse we ~~a~~ use the method of contrapositio
Let us assume n is even, we have to show $5n+6$ is even
Let n is even
$\Rightarrow n = 2k$ (where k is an even integer)
$\Rightarrow 5n+6 = 5(2k)+6 = 10k+6 = 2(5k+3)$
$= 2p$ (where $p = 5k+3$)
= even

∴ proved if n is positive integer then n is odd iff $5n+6$ is odd.

Recursion

Def^n: The process in which the function calls itself directly or indirectly is called recursion and the corresponding function is called as recursive function

- In recursion it uses two steps i.e basic step and second is recursive step.
- In basic step we specify some initial values and provide a rule for ~~contrad~~ constructing new elements from those we already have in the recursive step.

eg: ① Find out $F(1)$, $F(2)$, $F(3)$ & $F(4)$ where it is given $F(0) = 3$ and $F(n+1) = 2f(n)+3$

$f(n+1) = 2f(n) + 3$

at $n=0$

$f(1) = 2f(0) + 3$ [∵ $f(0) = 3$, given]

$= 2 \cdot 3 + 3$

$= 9$

$n=1$, $f(2) = 2f(1) + 3$

$= 18 + 3$

$= 21$

$n=2$ $f(3) = 2 \cdot f(2) + 3$

$= 2 \cdot 21 + 3$

$= 45$

$n=3$ $f(4) = 2 \cdot f(3) + 3$

$= 2 \cdot 45 + 3 = 93$

2. Find $f(2)$, $f(3)$, $f(4)$ & $f(5)$ if f is defined recursively $f(0) = -1$, & $f(1) = 2$ & for $n = 1, 2, \cdots$ $f(n+1) = f(n) + 3f(n-1)$

Sol^n: $f(2) = f(1) + 3f(0)$

$= 2 + 3(-1)$

$= -1$

$f(3) = f(2) + 3f(1)$

$= -1 + 3 \cdot 2 = -1 + 6 = 5$

$f(4) = f(3) + 3 \cdot f(2)$

$= 5 + 3 \cdot (-1)$

$= 5 - 3 = 2$

$f(5) = f(4) + 3 \cdot f(3)$

$= 2 + 3 \cdot 5$

$= 17$

③ Find $f(2)$, $f(3)$, $f(4)$, $f(5)$ if f is defined recursively by $f(0) = f(1) = 1$ & for $n = 1, 2, \ldots$ and $f(n+1) = f(n) - f(n-1)$

$n = 1$ $f(2) = f(1) - f(0)$

$= 1 - 1 = 0$

$n = 2$ $f(3) = f(2) - f(1)$

$= 0 - 1 = -1$

$n = 3$ $f(4) = f(3) - f(2)$

$= -1 - 0$

$= -1$

$n = 4$ $f(5) = f(4) - f(3)$

$= -1 - (-1)$

$= 0$

④ Give a recursive ~~function~~ definition of the sequence $\{a_n\}$ where $n = 1, 2, 3, \ldots n$ and if $a_n = 4n - 2$

soln: $a_n = 4n - 2$ (given)

at $n = 1$, $a_1 = 4(1) - 2 = 2$

$n = 2$, $a_2 = 4(2) - 2 = 6$

$n = 3$, $a_3 = 4(3) - 2 = 10$

$n = 4$, $a_4 = 4(4) - 2 = 14$

From the above sequences we can find out or develope a recursive function as

$$a_{n+1} = a_n + 4$$

⑤ Give a recursive definition of the sequence $\{a_n\}$ where $n = 1, 2, 3, \ldots$ & if $a_n = n(n+1)$

$n = 1, \ a_1 = 1(1+1) = 2$

$n = 2, \ a_2 = 2(2+1) = 6$

$n = 3, \ a_3 = 3(3+1) = 12$

$n = 4, \ a_4 = 4(4+1) = 20$

. . . .

. . . .

From the above sequence of series we can develope f^n recursively as $a_{n+1} = a_n \frac{n+2}{n}$

Structural Induction

- We can use mathematical induction over the set of +ve integers and recursive difinition to prove a result about a recursively difined sets. How ever instead of using mathematical induction directly to prove the results about reconsively defined sets we can use a more convinient form of induction known as structural induction.

Q. Show that $f_{n+1} f_{n-1} - f_n^2 = (-1)^2$ where n is a +ve integer

Let $P(n) = f_{n+1} f_{n-1} - f_n^2 = (-1)^2$

at $n = 1$ $\quad P(1) = f_2 f_0 - f_1^2 = (-1)^1$

LHS.

at $n = k$

$P(k) = f_{k+1} f_{k-1} - f_k^2 = (-1)^k$ ——①

Let us assume the above statement is true at $n = k$

at n = k+1 , $y_{k+2} y_k = (-1)^{k+1}$ — ②

we have to show the above statement is true at n = k+1 , $\boxed{y_{k+2} y_k - y_{k+1}^2 = (-1)^{k+1}}$

LHS. $y_{k+2} y_k - y_{k+1}^2$

$= (y_k + y_{k+1}) y_k - (y_{k+1} \cdot y_{k+1})$ [∵ $y_{k+2} = y_k + y_{k+1}$]

$= y_k^2 + y_k \cdot y_{k+1} - (y_k + y_{k-1}) \cdot y_{k+1}$ [∵ $y_{k+1} = y_k + y_{k-1}$]

$= y_k^2 + \cancel{y_k \cdot y_{k+1}} - \cancel{y_k \cdot y_{k+1}} - y_{k-1} \cdot y_{k+1}$

$= y_k^2 - y_{k-1} \cdot y_{k+1}$

$= (-1)(y_{k-1} \cdot y_{k+1} - y_k^2)$

$= (-1)(-1)^k$

$= \cancel{(-k)} (-1)^{k+1} =$ RHS.

SET

Set: A set can be defined as collection of unordered elements represented with in a curly braces ({}).

eg. Let us consider a set A which is a set of vowels $A = \{a, e, i, o, u\}$

B = set of even numbers

$B = \{2, 4, 6, 8, \ldots\}$

- The above presentation of set can be represented as Set Builder's Notation.

i.e $B = \{2, 4, 6, 8\}$ can be written as

$B = \{x \mid x \in \text{even no and } x \leq 8\}$

- The representation of some important predefined sets are.

N : Set of natural nos : $\{0, 1, 2, 3, 4 \ldots\ldots\}$

Z : set of Integers : $\{\ldots\ldots -2, -1, 0, 1, 2, \ldots\ldots\}$

Z^+ : a set of +ve integers : $\{1, 2, 3, \ldots\ldots\}$

Q : set of Rational Numbers :

$\{P/q, p \in Z, q, \in Z, q \neq 0\}$

R = set of real nos.

Types of Set :

Infinite Set : These types of sets can contain infinite no. of elements and the representation of this set can be defined as $A = \{5, 6, 7, \ldots\ldots\}$

$A : \{x \mid x \in N \text{ \& } x \geq 5\}$

(or) $A : \{x \mid x \in Z^+ \text{ \& } x \geq 5\}$

Finite Set : In this type of sets, elements are defined in a limited manner.

$A = \{5, 7, 8, 9\}$

$A : \{x \mid x \in N, 5 \leq x \leq 9\}$

Subset :

- If A is a subset of B then every element of A is an element of B ($A \subset B$)

$A = \{1,2\}$, $B = \{1,2,3,4\}$

$A \subset B$

$A \subseteq B = \{(x,y) \mid x \in A, y \in B, x \subseteq y\}$

Universal Set :

- It is a collection of all elements in a particular content i.e we can represent all ~~elements~~ animals in to a single set A is called universal set and can be represented as

$A = \{x \mid x \in \text{all animals}\}$

Empty set :

- This set can be represented as ϕ also called as NULL set which does not contain any elements and can be represented as $|A| = 0$

$A = \{\ \}$

$A = \{x \mid x \in N, 3 < x < 4\}$

Single term Set / Unit Set :

- A set containing only one elements is called unit set / single term set. $|A| = 1$
- Can be represented as

$A = \{4\}$

$A = \{x \mid x \in N, 3 < x < 5\}$

Equal Sets :

- Two sets A, B are said to be equal if all the elements of ~~a~~ A are present in B and vice versa.

$A = \{1,2\}$, $B = \{1,2\}$

represented as $A = B$ ($A \subseteq B$ & $B \subseteq A$)

$A = B$ $\{(x,y) \mid x \in A, y \in B \text{ and } x = y\}$

Equivalent Set :

- Two sets are said to be Equivalent if the number of elements in A = no. of. elements in B ($|A| = |B|$)

$A = \{1,2,3\}$, $B = \{6,2,9\}$

Disjoint Set :

- Two sets A and B are said to be disjoint if they don't have even one element as common

$A = \{1,2,3\}$, $B = \{6,7\}$

i.e. neither equal nor equivalent

Overlaping Set :

- Two sets are said to be overlaping set if they are having at least one element as common.

eg: $A = \{1,2,3\}$, $B = \{2,5,6\}$ ($A \cap B \neq \phi$, $|A \cap B| \neq 0$)

Cardinality of Set :

- It can be represented as $|A|$ where A is a set and it will return the no. of elements present in the set A. ($|A|$ = no. of elements in A)

Cartesian Product of Sets :

- Let A and B are two sets than the cartesian product of A & B can be denoted by $(A \times B)$ and can be defined by a set of order pairs

$A \times B = \{(a,b) \mid a \in A, b \in B\}$ ($|A \times B| = |A| \times |B|$)

Q. Find $A \times B$ where $A = \{1,2\}$, $B = \{3,4,5\}$

$A \times B = \{(1,3),(1,4),(1,5),(2,3),(2,4),(2,5)\}$

~~order pair~~

Q. Find $A \times B \times C$ where

$A = \{1,2\}$, $B = \{3,4,5\}$, $C = \{7,8,9\}$

$A \times B \times C = \{(1,3), (1,4), (1,5), (2,3), (2,4), (2,5)\} \times \{7, 8, 9\}$

$= ((1,3),7), ((1,3),8), ((1,3),9), ((1,4),7), ((1,4),8)$
$((1,4),9), ((1,5),7), ((1,5),8) ((1,5),9),$
$((2,3),7), ((2,3),8), ((2,3) 9)$
$((2,4),7), ((2,4),8), ((2,4),9)$
$((2,5),7), ((2,5),8), ((2,5),9)$

Q. Find out A×B×C i.e a set of order triplets
$A = \{1,2\}, B = \{6,3,4\}, C = \{5,7\}$

$A \times B \times C = \{(1,6,5), (1,6,7), (1,3,5), (1,3,7), (1,4,5), (1,4,7),$
$(2,6,5), (2,6,7), (2,3,5), (2,3,7), (2,4,5), (2,4,7)\}$

Ven Diagram

- It is the diagramatic representation of all logical relations between different sets and it was invented by John Ven in 1880.

Operation on Set :

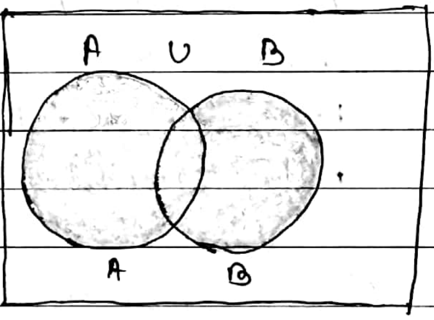
Let A = {1, 2, 3, 4} , B = {5, 6, 3, 2}, C = {7, 8, 9, 10}
the different operation on set are as follows

1. Union

the union operation of two sets A & B is represented as A ∪ B and can be defined as

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$

eg. $A \cup B = \{1, 2, 3, 4, 5, 6\}$

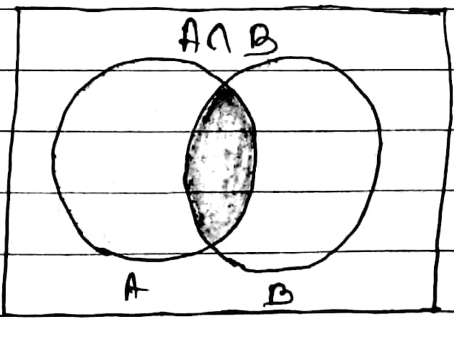


2. Intersection

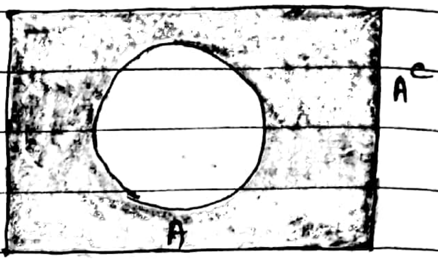
- The intersection of two sets A & B can be represented as A ∩ B and can be defined as

$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$

eg. $A \cap B = \{2, 3\}$



3. Complement of a Set



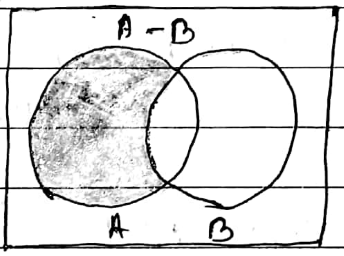
- Complement of set A can be represented as " A^c " or " $\bar{A}$ " and can be defined as

$A^c = \{x / x \notin A\}$

eg: Let $x = \{1, 2, 3 \ldots\ldots 10\}$

Let us Assume set A is defined from set x then $A^c = \{5,6,7,8,9,10\}$ where $A = \{1,2,3,4\}$

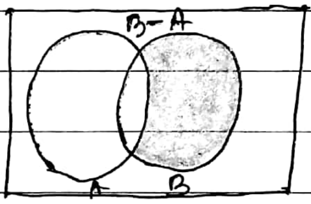
4. Difference



- The difference of two sets A & B can be represented as "A - B" or "A \ B" and can be defined as:

$A - B = \{x / x \in A \text{ and } x \notin B\}$

$A - B = \{1, 4\}$



Now consider "B-A" or "B \ A"

$B - A = \{x / x \in B \text{ and } x \notin A\}$

$B - A = \{5, 6\}$

6. Proof $\overline{A \cap B} = \bar{A} \cup \bar{B}$ using set builder's notation and logical equivalances.

ans: $\overline{A \cap B} = \bar{A} \cup \bar{B}$

$\overline{A \cap B} = \{x / x \notin A \cap B\}$ [∵ notations]

$= \{x / \sim(x \in A \cap B)\}$ [Rules of negation on belongs to]

$= \{x / \sim x \in A \cup \sim x \in B\}$ [∵ Demorgans]

$= \{x / x \notin A \cup x \notin B\}$ [∵ Rule of negation on belongs to]

$= \bar{A} \cup \bar{B}$

Q. Prove that $\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$
using setbuilder's notation as well as logical equivalance

Proof: using logical equivalances
LHS: $\overline{A \cup (B \cap C)}$
$= \overline{A} \cap \overline{(B \cap C)}$ [∵ Demorgan's Law]
$= \overline{(B \cap C)} \cap \overline{A}$ [∵ Commutative Law]
$= (\overline{B} \cup \overline{C}) \cap \overline{A}$ [∵ Demorgan's Law]
$= (\overline{C} \cup \overline{B}) \cap \overline{A}$ = RHS [∵ commutatie Law]

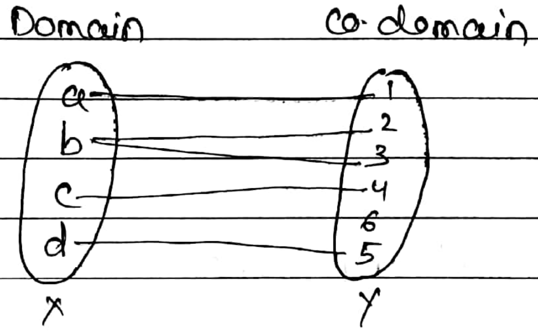
Using SetBuilder's Notation
LHS = $\overline{A \cup (B \cap C)} = \{x | x \notin A \cup (B \cap C)\}$ ∵ Notation on Rule of negation
$= \{x | \sim (x \in A \cup (B \cap C))\}$ ∵ Rule of negation on belongs to
$= \{x | \sim (x \in (B \cap C) \cup A)\}$ ∵ Commutative
$= \{x | \sim (x \in (B \cap C)) \cap \sim x \in A]$ ∵ Demorgan's Law
$= [x | (\sim x \in B \cup \sim x \in C) \cap \sim x \in A]$ ∵ Demorgan's Law
$= \{x | (\sim x \in C \cup \sim x \in B) \cap \sim x \in A]$ ∵ commutative Law
$= \{x | (x \notin C \cup x \notin B) \cap x \notin A\}$ ∵ Rule of negation on belongs to
$= (\overline{C} \cup \overline{B}) \cap \overline{A}$ = RHS

Q. Prove using set Builder's Notation
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
LHS: $A \cup (B \cap C)$
$= \{x | x \in A \cup (B \cap C)]$ ∵ using notation
$= \{x | x \in ((A \cup B) \cap (A \cup C))\}$ ∵ Distributive Law
$= [x | x \in (A \cup B) \cap x \in (A \cup C)]$ ∵ Notation $= (A \cup B) \cap (A \cup C)$ = RHS

-: Function :-

· Functions are sometimes called mappings or transformations.

(i) It is a relationship from one set to another suppose X and Y are two sets then the function / mapping from X to Y can be represented as f : X → Y

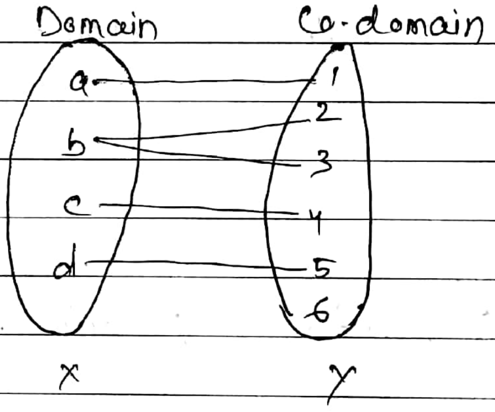


(ii) Here X represents the domain of the function and Y represents co-domain of the function

here X = {a, b, c, d}

Y = {1, 2, 3, 4, 5, 6}

Range of X = {1, 2, 3, 4, 5}

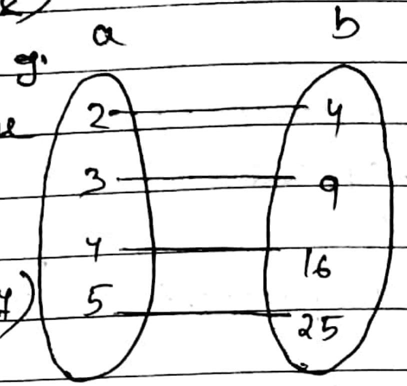


Types of Functions

- Function can be catagorised in to 4 types
  1. One-one (Injective)
  2. Many-One
  3. On-to (Surjective)
  4. Bijective

1. One - One (Injective)

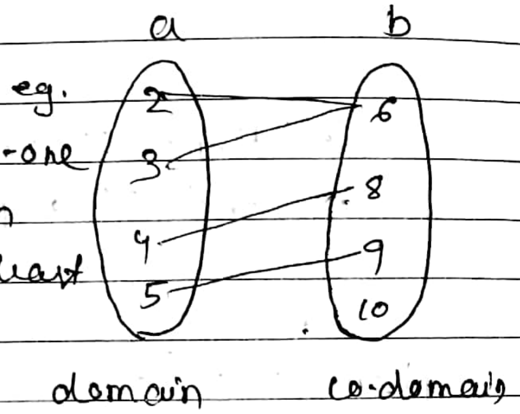
- A Function f is said to be one-one or injective if f(a) = f(b) => a = b (for all a & b are the domain of f)



(one - one)

2. Many - One

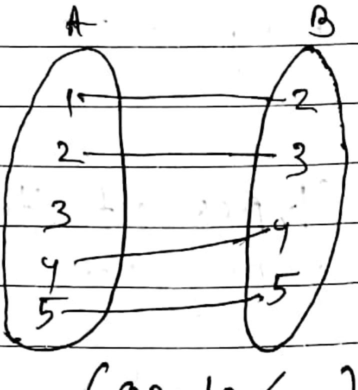
- A function is said to be many-one if more than one element from domain are linked with at least one from co domain



(many - one)

3. On-to (Surjective)

- A Function f : A→B is said to be On-to or Surjective if every element of B is an image of some elements of A

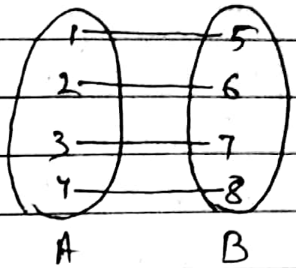


(On-to / Surjective)

4. Bijective

- A function $f: A \to B$ is said to be bijective if it is both injective and surjective

$f: A \to B$



Q. Prove that $f: N \to N$ where it is given that $f(x) = 2x$ is not Bijective

Soln: Given $f(x) = 2x$

Let us consider $f(x_1) = f(x_2)$

$\Rightarrow 2x_1 = 2x_2$

$\Rightarrow x_1 = x_2$

$\Rightarrow$ one-one

again to show wheather surjective or not

$f(x) = 2x, \ x \in N, \ y \in N$

$\Rightarrow y = 2x$

$\Rightarrow x = y/2 \Rightarrow 1/2 \notin N$

Here if $y = 1$ then $x = 1/2$ which is not a natural no so the function is not on-to or surjective.

∴ As the function is One-One but not On-to the function is not Bijective.

Q. Prove that $f: R \to R$, $f(x) = 1/x$ show it is a bijective

Soln: one-one : Given $f(x) = 1/x$

Let us consider $f(x_1) = f(x_2)$

$\Rightarrow 1/x_1 = 1/x_2$

$\Rightarrow x_1 = x_2$

$\Rightarrow$ one-one

On-to :

$f(x) : y = 1/x$

$\Rightarrow x = 1/y \; (1/y \in R)$

$\Rightarrow$ on-to

As the function is both injective & serjective so the function is bijective

Q. Prove that $f: Z \to Z$, $f(x) = x^2$ is a bijective

proof:

Given $f(x) = x^2$

One-one

Let us consider $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = \pm x_2$

$\Rightarrow$ not one-one

On-to

$f(x) = y = x^2$

$\Rightarrow x = \pm\sqrt{y} \notin Z$

here there is no such integer $x \in Z$

$\Rightarrow f(x) = x^2 \notin -3, -5, \ldots$

Composition of function

- Let $f: A \to B$ and $f: B \to C$ are two functions then its composition can be defined as a function $f: A \to C$ and can be represented as

$$gof(x) = g(f(x)) \text{ (or)}$$
$$fog(x) = f(g(x))$$

Let $f(x) = x+2$ & $g(x) = 2x$ find

$fog(x)$ & $gof(x)$

$f(x) = x+2$, $g(x) = 2x$

$fog(x) = f(g(x))$
$= f(2x)$
$= 2x+2$

$gof(x) : g(f(x))$
$= g(x+2)$
$= 2(x+2)$
$= 2x+4$

Q. Find out gof & fog
a) $f(x) = |x|$ and $g(x) = |5x-2|$
b) $f(x) = 8x^3$ and $g(x) = x^{1/3}$

a) $fog(x) = f(g(x))$
$= f(|5x-2|)$
$= |5x-2| = 5x-2$

$gof(x) = g(f(x))$
$= g(x)$
$= |5|x|-2|$
$= |5x-2| = 5x-2$

b) $fog(x) = f(g(x))$
$= f(x^{1/3})$
$= 8(x^{1/3})^3$
$= 8x$

$gof(x) = g(f(x))$
$= g(8x^3)$
$= (2^3 x^3)^{1/3}$
$= 2x$

Q/ Let $f: \{1,3,4\} \to \{1,2,5\}$

$g: \{1,2,3\} \to \{1,3\}$

$f: \{(1,2),(3,5),(4,1)\}$ & $g = \{(1,3),(2,3),(5,1)\}$

Ans: Given $f = \{(1,2),(3,5),(4,1)\}$

$f(1) = 2$

$f(3) = 5$

$f(4) = 1$

Given $g = \{(1,3),(2,3),(5,1)\}$

$g(1) = 3$

$g(2) = 3$

$g(5) = 1$

$$fog(x) = f(g(x))$$
$$= f(g(1))$$
$$= f(3)$$
$$= 5$$

& $fog(g(x)) = f(g(2))$
$$= f(3)$$
$$= 5$$

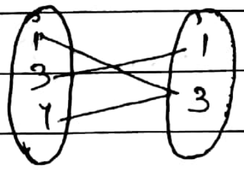
$$fog(g(x)) = f(g(5))$$
$$= f(1)$$
$$= 2$$

$$gof(f(x)) = g(f(x))$$
$$= g(f(x))$$
$$= g(f(1))$$
$$= g(2)$$
$$= 3$$

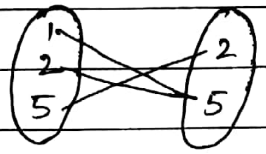
& $gof(f(x)) = g(f(3))$
$$= g(5) = 1$$

$g(f(x)) = g(f(4))$
$= g(1)$
$= 3$

The representation of fog & gof as



gof



(fog)

Q/ $f: R \to R$, $g: R \to R$, $h: R \to R$
ST: a) $(f+g)oh = foh + gof$
b) $(fg)oh = (foh) \cdot (goh)$

Ans: a) LHS
$(f+g)oh$
$= (f+g)(h(x))$
$= f(h(x)) + g(h(x))$
$= foh + goh =$ RHS

b) LHS
$(fg)oh$
$= (fg)(h(x))$
$= f(h(x))g(h(x))$
$= foh \cdot goh =$ RHS

Q. Let f(x) $= \frac{4x+3}{6x-4}$, $x \neq 2/3$

ST. $fof(x) = x$ for all $x \neq 2/3$ and check inventible

Ans: Given $f(x) = \frac{4x+3}{6x-4}$

$$fof(x) = f(f(x)) = \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}$$

$$= \frac{16x + \cancel{12} + 18x - \cancel{12}}{\cancel{24x} + 18 - \cancel{24x} + 16}$$

$$= \frac{34x}{34} = x = RHS$$

To check the function is inventible we have to follow the following steps.

Stp-1

We have to represent $f(x)$ in terms of y and name it as $g(y)$

Let $f(x) = y = \frac{4x+3}{6x-4}$

$$\Rightarrow 6xy - 4y = 4x+3$$

$$\Rightarrow 6xy - 4x = 4y+3$$

$$\Rightarrow x(6y-4) = 4y+3$$

$$\Rightarrow x = \frac{4y+3}{6y-4} = g(y)$$

Stp-2

We have to show $fog(y) = y$

LHS

$$fog(y) = f(g(y)) = \frac{4\left(\frac{3+4y}{6y-4}\right)+3}{6\left(\frac{3+4y}{6y-4}\right)-4}$$

$$= \frac{\cancel{12} + 16y + 18y - \cancel{12}}{18 + \cancel{24y} - \cancel{24y} + 16}$$

$$= \frac{34y}{34} = y = RHS.$$

Stp-3

We have to show $gof(x) = x$

LHS

$$gof(x) = g(f(x))$$

$$= \frac{3+4\left(\frac{4x+3}{6x-4}\right)}{6\left(\frac{4x+3}{6x-4}\right)-4}$$

$$= \frac{18x-12+16x+12}{24x+18-24x+16}$$

$$= \frac{24x}{34} = x$$

On considering the above case it concluded that the function is inventible.

Q. $f: [-1,1] \to R$ and $f(x) = \frac{x}{x+2}$

show that $f(x)$ is one-one and find its inverse

(check inventible)

Ans: One-one

$f(x_1) = f(x_1)$

$\Rightarrow \frac{x_1}{x_1+2} = \frac{x_2}{x_2+2}$

$\Rightarrow x_1x_2 + 2x_1 = x_1x_2 + 2x_2$

$\Rightarrow 2x_1 = 2x_2$

$\Rightarrow 2x_1 = 2x_2$

$\Rightarrow x_1 = x_2$ one-one preved

Stp-1

Let us consider $y = \frac{x}{x+2}$

$\Rightarrow xy + 2y = x$

$\Rightarrow 2y = x - xy$

$\Rightarrow x(1-y) = 2y$

$\Rightarrow x = \frac{2y}{1-y} = g(y)$

Stp.2

We have to show fog $(y) = y$

$$fog\,(y) = f(g(y))$$

$$= \frac{2y/1-y}{\frac{2y}{1-y}+2} = \frac{2y}{2y+2-2y} = y = RHS.$$

Stp.3

We have to show

$$gof\,(x) = x$$

$$gof\,(x) = g\,(f(x))$$

$$= \frac{2\left(\frac{x}{x+2}\right)}{1-\left(\frac{x}{x-2}\right)}$$

$$= \frac{2x/x+2}{\frac{x+2-x}{x+2}} = \frac{2x}{2} = x = RHS.$$

On considering the above cases it concluded that the Function is inventible and also the Functions are inverse to each other.

(proved)

Relation :

Let 'A' & 'B' are two sets then the relations are from Set A & B is a subset of the cartesian product A × B i.e $R = A \times B = \{(x,y) \mid x \in A, y \in B\}$

Let $A = \{1,2,3\}$, & $B = \{3,5\}$ then find out the relational matrix where $R = \{(x,y) \mid x \text{ and } y \in \text{odd nos}\}$

ans: $A = \{1,2,3\}$, $B = \{3,5\}$

$A \times B = \{(1,3), (1,5), (2,3), (2,5), (3,3), (3,5)\}$

$R = \{(1,3), (1,5), (3,3), (3,5)\}$

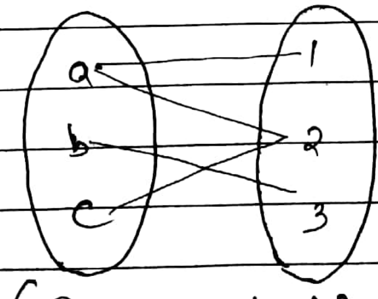
Now relational matrix

$$\begin{array}{c|cc} & 3 & 5 \\ \hline 1 & 1 & 1 \\ 2 & 0 & 0 \\ 3 & 1 & 1 \end{array}$$

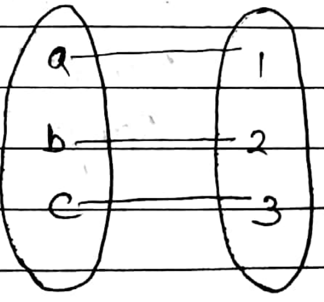
Relation vs Function :

- A relation can be ~~expressed~~ used to express one-many relationships between the set A & the element of B where as in function, it does not support one-many relationship.
- The function may be treated as relation but not vice versa.

eg.



(Relation but not a function)



(Relation as well as function)

Binary Relation

- Can be defined as relation of elements of two sets A & B and if the relationship is between more than two sets then relation is called n-array relation.

eg. Database relation is a n-array relation. and the above relation are binary relation.

Domain and Range of Relation:

- The domain of a range can be defined as Dom(R) Range can be defined as Range(R)

- Domain is a set of all 1st elements of the ordered pairs who belongs to R and Range is a set of 2nd elements of the ordered pairs who belongs to R

Q. Find out domain and range of the relation

$R = \{(x,y) \mid y = x+1, x \in A, y \in B\}$

where $A = \{1,2,3\}$, $B =$ ~~[illegible]~~ $\{3,4,5\}$

Soln: Given $A = \{1,2,3\}$, $B = \{3,4,5\}$

$A \times B = \{(1,3),(1,4),(1,5),(2,3),(2,4),(2,5),(3,3),(3,4),(3,5)\}$

Now $R = \{(2,3),(3,4)\}$

$Dom(R) = \{2,3\}$

$Range(R)$:

Types of Relation :

* **Inverse Relation :**

- If R is a relation then inverse of the relation can be denoted as 'R^{-1}'.
- If R is a relation from set A to B then it is said to be as inverse relation if it contains the ordered pairs i.e from set B to A

i.e $R^{-1} = \{(b,a) / (a,b) \in R\}$

Q. Find out the inverse relation $R = \{(x,y) | x<y, x \in A, x \in B\}$

where $A = \{1,2,3\}$, $B = \{3,4\}$

$A \times B = \{(1,3), (1,4), (2,3), (2,4), (3,3), (3,4)\}$

$R = \{(1,3), (1,4), (2,3), (2,4), (3,4)\}$

$R^{-1} = \{(3,1), (4,1), (3,2), (4,2), (4,3)\}$

* **Identity Relation**

- A relation in a set A is said to be identity when

$I_A = \{(x,x) | x \in A\}$

eg : $A = \{1,2,3\}$

$R = \{(x,y) | x, y \in A\}$

$A \times A = \{1,2,3\} \times \{1,2,3\}$

$= \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

$I_A = \{(1,1), (2,2), (3,3)\}$

* **Operation on Relation :**

Let R_1 & R_2 be two relations defined on the set

$A = \{1,2,3\}$ & $B = \{1,2,3,4\}$

& $R_1 = \{(1,1), (2,2), (3,1)\}$

$R_2 = \{(1,1), (1,3), (1,4), (1,2)\}$

Then different operations on relation are

1. Intersection $(R_1 \cap R_2)$

$$R_1 \cap R_2 = \{(1,1)\}$$

2. Union $(R_1 \cup R_2)$

$$R_1 \cup R_2 = \{(1,1)(1,3)(1,4)(1,2)(2,2)(3,3)\}$$

3. Difference

$$R_1 = \{(1,1)(2,2)(3,3)\}$$
$$R_2 = \{(1,1),(1,3),(1,4),(1,2)\}$$
$$R_1 - R_2 = \{(2,2),(3,3)\}$$
$$R_2 - R_1 = \{(1,3),(1,4),(1,2)\}$$

4. Exclusive OR $(R_1 \oplus R_2)$

$$R_1 \oplus R_2 = (R_1 \cup R_2) - (R_1 \cap R_2)$$
$$= \{(1,1),(1,3),(1,4),(1,2),(2,2),(3,3)\} - \{(1,1)\}$$
$$= \{(1,3),(1,4),(1,2),(2,2),(3,3)\}$$

Properties of Relation

* Reflexive :

A relation R on a set A is called reflexive if $(a,a) \in A$ $\forall a \in A$

eg: Let us consider the relation defined on $A = \{1,2,3,4\}$

$R_1 = \{(1,2),(1,3),(4,2),(4,3),(3,3)\}$ is not reflexive

and $R_2 = \{(1,2),(4,2),(1,1),(2,3),(2,2),(3,1),(3,3),(4,2),(4,4)\}$ is

R_1 is not reflexive because $(2,2),(1,1),(4,4) \notin R$

R_2 is reflexive because $(1,1),(2,2),(3,3),(4,4) \in R$

* Symetric Relation :

The relation R on set A is called symetric if $(b,a) \in R$ whenever $(a,b) \in R$ for all $(a,b) \in A$

eg: $R = \{(1,2),(2,2),(2,1),(3,1),(1,3)\}$ is a symetric

$R = \{(1,2),(3,3),(4,4),(2,1)\}$ is not-symetric

Antisymetric :

- A relation R is said to be antisymetric if there is no pair of elements (a, b) with $a \neq b$ such that (a,b) & (b, a) belongs to the relation.

$R = \{(1,2), (2,2), (2,1), (3,2)\}$ is an antisymetric

$R = \{(1,2), (3,2), (4,3)\}$ is antisymetric

Transitive

A relation R on a set A is said to be transitive if, whenever (a, b) $\in$ R &
(b, c) $\in$ R then
(a, c) $\in$ R $\forall$ (a, b, c) $\in$ A

$R = \{(1,2), (2,1), (1,1), (3,4), (4,2), (3,2)\}$ ~~it~~ it is a transitive

Q. Wheather the following relation is reflexive, symetric or transitive where $A = \{1,2,3,4\}$
$R = \{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)\}$

Reflexive : Defn :- ———
As (1,1), (2,2), (3,3), (4,4) $\in$ R
so the relation is reflexive

Symetric : Defn : ———
For all (a, b) $\in$ R : there is (b, a) $\in$ R so the relation is symetric

Transitive : Defn : ———
For all (a, b) $\in$ R & (b, c) $\in$ R there is (a, c) $\in$ R

Composition of Relation :

- Let R be a relation from set A→B & S be a relation from B→C then the composition of R→S can be represented as SoR, can be defined as

$$SoR = \{(a,c) \mid (a,b) \in R \text{ and } (b,c) \in S\}$$

eg. Find out the composition of the relations R & S where R is a relation from {1,2,3} to {1,2,3,4} & S is a relation from {1,2,3,4} to {0,1,2} with

$R = \{(1,1), (1,4), (2,3), (3,1)\}$ &
$S = \{(1,0), (2,0), (3,1), (3,2), (4,1)\}$

Soln: $SoR \{(1,0), (1,1), (2,1), (2,2), (3,0)\}$

Q. Let R & S be two relations where
$R = \{(1,2), (1,3), (2,3), (2,4), (3,1)\}$
$S = \{(2,1), (3,1), (3,2), (4,2)\}$

Find SoR
Soln: $SoR = \{(1,1), (1,2), (2,3), (2,2)\}$

Representation of Relation :

The relation can be represented in two ways
1. By using matrix
2. By using diagraphs.

1. By using Matrix

- If R is a relation then its matrix representation can be denoted as "M_R" and if "m_{ij}" is a element of matrix M_R then it can be defined as

$$m_{ij} = \begin{cases} 1, \text{ if } (a_i, b_j) \in R \\ 0, \text{ if } (a_i, b_j) \notin R \end{cases}$$

Q. Find out the relational matrix of the relation
R: [(1,2), (2,1), (3,2), (4,3)]
defined on set A: [1, 2, 3, 4].

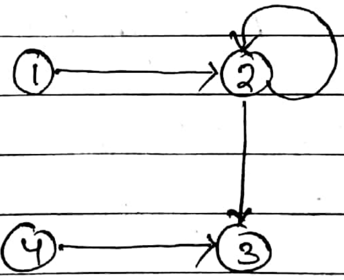
$$M_R:\quad \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

2. By Using Diographs:

The diograph can be consists of a set of vertices and a set of edges (E) and a set of ordered pairs s.t (a,b) ∈ R if there is a edge from a→b where

$a \in A$ and $b \in B$

eg: Represent the relation defined on a set
A = [1,2,3,4]
R: [(1,2),(2,2),(2,3),(4,3)]



Let R_1 and R_2 is the relation on set A represented by

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \qquad M_{R_2} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

then $R_1 \cup R_2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ $\begin{Bmatrix} \max(1,0) = 1 \\ \max(0,1) = 1 \end{Bmatrix}$

$R_1 \cap R_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $\{\because \min(1,0) = 0\}$

$R_1 \odot R_2 = (\max(\min)) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

Equivalance Relation

A relation R on set A is said to be equivalance if R is reflexive, symetric and transitive.

eg. Let $A = \{1, 2, 3\}$

$R_1 = \{\ \}$ × not reflexive

$R_2 = \{(1,1), (2,2), (3,3)\}$ ✓

(reflexive, symetric, transitive)

$R_3 = \{(1,1), (2,3), (3,3), (3,1)\}$

(reflexive, not symetric, transitive)

$R_4 = \{(1,1), (2,2), (2,1), (1,2)\}$

(not reflexive, symetric, transitive)

Partial Order Relation

- A relation R on a set A is said to be partial order relation iff it is reflexive, antisymetric and transitive

$R_1 : [\ ]$ × bcoz not reflexive

$R_2 = \{(1,1), (2,2), (3,3)\}$ ✓

$R_3 = \{(1,1), (1,2), (3,2), (3,3)\}$ × because not reflexive

Asymetric	Antisymetric
1. Relation R is said to be asymetric on a set A if for all $(a,b) \in R$ then $(b,a) \notin R$.	1. A relation R on set A is said to be antisymetric ~~if~~ $\forall (a,b) \in A$, if $(a,b) \in R$ then $(b,a) \in R$ iff $a = b$
eg: $R = \{\ \}$ ✓ $R = \{(1,2), (3,1), (3,2)\}$ ✓	eg: $R : \{\ \}$ ✓ $R = [(1,1), (2,2), (3,3)]$ ✓ $R = \{(1,1), (2,1), (3,1)\}$ $R = \{(1,1), (2,1), (3,1)\}$ (Anty symetric but not asymetric)

Closure of Relation

- Let R be a relation on set A and that may or may not have same properties "P" such as Reflexive, Symetric, or transitive

- If there is relation S with "P" containing R such that 'S' is a subset of every relation with property "P" containing 'R' then 'S' is called the closure of R with respect to "P"

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Reflexive Closure :

- Let R is a relation on the set of integers then the reflexive closure of R can be defined as "R ∪ Δ" = $\{(a,b) \mid a<b\} \cup \{(a,a) \mid a \in A\}$

eg: Let $A = \{1,2,3\}$
$R = \{(a,b) \mid a<b\}$
then $R = \{(1,2), (2,3), (1,3)\}$
the reflexive closure of the above relation is
$R \cup \Delta = \{(1,2), (2,3), (1,3)\} \cup \{(1,1), (2,2), (3,3)\}$
$= \{(1,2), (2,3), (1,3), (1,1), (2,2), (3,3)\}$

Symetric Closure :

The symetric closure of a relation R can be represented as $R \cup R^{-1}$ and can be defined as
$R \cup R^{1} = \{(a,b) \mid a<b\} \cup \{(b,a) \mid a,b \in A\}$

the symetric closure of the above relation is
$R \cup R^{-1} \{(1,2), (2,3), (1,3)\} \cup \{(2,1), (3,2), (3,1)\}$
$= \{(1,2), (2,3), (1,3), (2,1), (3,2), (3,1)\}$

Transitive Closure :

- The transitive closure of relation R can be represented as $R \cup R^2 \cup R^3 \cup \cdots R^n$ where n = no of elements present on set A.

and $R^2 = R \circ R$
$R^3 = R^2 \circ R$
. . . .

eg: In the example
$A = \{1,2,3\}$
where $n = 3$
then transitive closure of R can be defined as

$R = R \cup R^2 \cup R^3$

$R = \{(1,2), (1,3), (2,3)\}$

$R^2 = \{ R \circ R = \{(1,2),(1,3),(2,3)\} \circ \{(1,2),(1,3),(2,3)\}$

$= \{(1,3)\}$

$R^3 = R^2 \circ R = \{(1,3)\} \circ \{(1,2),(1,3),(2,3)\}$

$= \{ \}$ (Empty set / NULL set)

$R = R \cup R^2 \cup R^3$

$= \{(1,2),(1,3),(2,3)\} \cup \{(1,3)\} \cup \{ \}$

$= \{(1,2),(1,3),(2,3)\}$

Q. Find the reflexive, symetric, transitive closcure of the relation $R = \{(1,2),(1,3),(3,1)\}$ defined on the set $A = \{1,2,3\}$

(i) the reflexive closcure of $R = R \cup \Delta$

$= \{(1,2),(1,3),(3,1)\} \cup \{(1,1),(2,2),(3,3)\}$

$= \{(1,2),(1,3),(3,1),(1,1),(2,2),(3,3)\}$

(ii) Symetric Closcure of R =

$R \cup R' = \{(1,2),(1,3),(3,1)\} \cup \{(2,1),(3,1),(1,3)\}$

$= \{(1,2),(2,1),(1,3),(3,1)\}$

(iii) Transitive Closure of $R = R \cup R^2 \cup R^3$

$R = \{(1,2),(1,3),(3,1)\}$

$R^2 = R \circ R = \{(1,2),(1,3),(3,1)\} \circ \{(1,2),(1,3),(3,1)\}$

$= \{(3,2)\}$

$R^3 = \{(3,2)\} \circ \{(1,2),(1,3),(3,1)\}$

$= \{ \}$

$R = R \cup R^2 \cup R^3$

$= \{(1,2),(1,3),(3,1)\} \circ \{(3,2)\} \circ \{ \}$

$= \{(1,2),(1,3),(3,1),(3,2)\}$

Irreflexive Relation :

- A set is said to be irreflexive on a set A then $\forall x \in R.\ (x, x) \in R$

eg. $R = \{\ \}$ ✓ Irreflexive

$R = A \times A$ ✗

Q. Show that $a = b \bmod 7$ is an equivalent relation on Z.

Solⁿ: $a = b \bmod 7$

$\Rightarrow a - b$ is divisible by 7

$\Rightarrow a - b = 7k$

} Definition of modulo division.

A relation is said to be equivalance if it is reflexive, symetric, transitive

Reflexive

Let $a = b$

Given $a = b \bmod 7$

$\Rightarrow a - b$ divisible by 7

$\Rightarrow a - a =$ divisible by 7 $[\because a = b]$

$\Rightarrow a = a \bmod 7$ = reflexive

Symetric

Given $a = b \bmod 7$

$\Rightarrow a - b$ is divisible by 7

$\Rightarrow a - b = 7k$

$\Rightarrow b - a = -7k$

$\Rightarrow b - a = 7(-k)$

$\Rightarrow b - a = 7k_1$ $[\because -k \in Z]$

$\Rightarrow b - a$ is divisible by 7

$\Rightarrow b = a \bmod 7$ = symetric

Transitive

Given $a = b \bmod 7$

$\Rightarrow a-b = 7k_1$ —— ①

Similarly we can write

$b = c \bmod 7$

$\Rightarrow b-c = 7k_2$ —— ②

adding equn ① & ② we get

$a-b+b-c = 7k_1 + 7k_2$

$\Rightarrow a-c = 7(k_1+k_2)$

$\Rightarrow a-c = 7k_3 \quad (\because k_1+k_2 \in Z)$

$\Rightarrow a-c$ is divisible by 7

$\Rightarrow a = c \bmod 7$ = Transitive

Q. Find out the matrix of transitive closure of the relation R to A where

$$M_R = \begin{Bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{Bmatrix}$$

The transitive closure of the above relational matrix can be defined as

$$M_R^* = ~~B~~ M_R \vee M_R^2 \vee M_R^3$$

$$M_R^2 = M_R \vee M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$M_R^3 = M_R^2 \vee M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

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$M_R^* = M_R \vee M_R^2 \vee M_R^3$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Boolean Algebra

- Boolean algebra is an algebra of Logic i.e it is one of the most basic tools to analyse and design logic circuits.
- The concept of boolean algebra was de~~p~~veloped by George Boole in 1954.
- Here we can represent the statement
  "Ram is eating an apple" in to "x" i.e
  x = Ram is eating an apple.

- Here x may be 'one' if it is true OR
  x may be 'zero' if it is false.

Operations on Boolean Algebra

- In boolean algebra the following operations

1. AND operations (x · y)

x	y	x · y
1	1	1
1	0	0
0	1	0
0	0	0

2. OR operation (x + y) :

x	x	x + y
1	1	1
1	0	1
0	1	1
0	0	0

3. NOT operation ($\bar{x}$)

x	$\bar{x}$
1	0
0	1

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Boolean Theorems

On the basis of AND operations

$0 \cdot x = 0$

$x \cdot 0 = 0$

$1 \cdot x = x$

$x \cdot 1 = x$

On the basis of OR operations

$x + 0 = x$

$0 + x = X$

$x + 1 = 1$

$1 + x = 1$

$x \cdot x = x$ $\qquad \overline{\overline{x}} = x$

$x \cdot \overline{x} = 0$ $\qquad x + y = y + x$

$x \cdot x = y$ $\qquad x \cdot y = y \cdot x$

$x + \overline{x} = 1$

$$\left.\begin{array}{l} x + xy = x \\ x(x+y) = x \\ xy + x\overline{y} = x \\ (x+y)(x+\overline{y}) = x \end{array}\right\} \text{absorption}$$

Prove

$(x+y)(x+\bar{y}) = x$.

LHS:

$(x+y)(x+\bar{y})$

$= xx + x\bar{y} + yx + y\bar{y}$

$= xx + x(\bar{y}+y) + 0$ (By theorem)

$= xx + x\cdot 1 + 0$ (By theorem)

$= xx + x$ (By theorem)

$= x(x+1)$

$= x\cdot 1$ (By theorem)

$= x$

By using truth table

x	y	$x+y$	$\bar{y}$	$x+\bar{y}$	$(x+y)\cdot(x+\bar{y})$
1	1	1	0	1	1
1	0	1	1	1	1
0	1	1	0	0	0
0	0	0	1	1	0

Equivalent
(LHS = RHS)

Show that $xy + \bar{x}z + yz = xy + \bar{x}z$

LHS: $xy + \bar{x}z + yz(x+\bar{x})$ [By theorem $x+\bar{x}=1$]

$= xy + \bar{x}z + yzx + yz\bar{x}$

$= xy(z+1) + \bar{x}z + yz\bar{x}$

$= xy\cdot 1 + \bar{x}z + yz\bar{x}$ {By theorem $z+1=1$}

$= xy + \bar{x}z(y+1)$ {By theorem $y+1=1$}

$= xy + \bar{x}z\cdot 1$

$= xy + \bar{x}z$ = RHS

Simplification of Boolean Expression

Q. $X + \bar{X}Y + \bar{Y} + (X+Y)\bar{X}Y$

$= X + \bar{X}Y + \bar{Y} + X\bar{X}Y + \bar{X}\bar{X}Y$

$= X + \bar{X}Y + \bar{Y} + 0 + 0$ {By theorem $X\cdot\bar{X} = 0$, $Y\cdot\bar{Y} = 0$}

$= X + \bar{X}Y + \bar{Y}$

Q. $Z(Y+Z)(X+Y+Z)$

$\Rightarrow (YZ + ZZ)(X+Y+Z)$

$\Rightarrow (YZ + Z)(X+Y+Z)$ (By them $Z.Z = Z$)

$\Rightarrow Z(Y+1)(X+Y+Z)$

$\Rightarrow Z.1(X+Y+Z)$ (By theorem $Y+1 = 1$)

$\Rightarrow Z(X+Y+Z)$ (By theorem $Z.1 = Z$)

$\Rightarrow ZX + ZY + Z.Z$

$\Rightarrow ZX + ZY + Z$ (By theorem $Z.Z = Z$)

$\Rightarrow ZX + Z(Y+1)$

$\Rightarrow ZX + Z(Y+1)$

$\Rightarrow ZX + Z.1$ (By theorem $Y+1 = 1$)

$\Rightarrow ZX + Z$ (By theorem $Z.1 = 1$)

$\Rightarrow Z(X+1)$

$\Rightarrow Z.1$ (By theorem $X+1 = 1$)

$\Rightarrow Z.1$

$\Rightarrow Z$ (By theorem $Z.1 = Z$)

Dual and Complement Expression of Boolean Expression

- Two expression are complement to each other if one expression = 1 and other expression = 0 & vice-versa

- To obtain the complement of boolean expression the following changes are made.

all " . " sign will be changed to " + " sign
all " + " sign will be change to " . " sign

- All zeros will be changed to one and all one will be changed to zero.
- All literals must be complemented

eg. the complement of the expression
$1 . X + \bar{Y} . Z + 0 . Z$
$= (0 . \bar{X}) . (Y + \bar{Z}) . (1 + \bar{Z})$

Duality of Boolean Expression

- The dual of a boolean expression can be obtained by performing the following operations

1. all " . " signs will be converted to " + " sign and vice-versa
2. all " 1 " will be converted to " 0 " and vice versa

eg. $1 . X + \bar{Y} . Z + 0 . Z$
$(0 + \bar{X}) . (Y + \bar{Z}) . (1 + \bar{Z})$

Find the duality of

$x(y+z) = xy + xz$

$\Rightarrow x + (y \cdot z) = (x+y) \cdot (x+z)$

Find dual of the expression and solve it

$x(\bar{y} + yz) + y\bar{z}$

$= [x + (\bar{y} \cdot yz)] \cdot (y + \bar{z})$

$= (x + \bar{y}y + \bar{y}z) \cdot (y + \bar{z})$

$= (x + 0 + \bar{y}z) \cdot (y + \bar{z})$ [∵ By theorem $\bar{y} \cdot y = 0$]

$= xy + x\bar{z} + \bar{y}yz + \bar{y}z\bar{z}$

$= xy + x\bar{z}$ [∵ By theorem $\bar{y} \cdot y = 0$, $z \cdot \bar{z} = 0$]

Sum of Products and Product of Sum of Logic Expression

- A logic expression is said to be sum of products if it is defined as "Sum of the product of the Literals" i.e SOP

$xy + x\bar{y} + xyz + \dots + \dots$

- A logical ~~sum~~ expression is said to be product of sum or POS if it is defined as "Product of literals"

$(x+y) \cdot (x+\bar{y}) \cdot (x+y+z) \dots\dots$

Scanned by CamScanner

Canonical Form of Logic Expressions

- A logical expression is said to be canonical form if each term of the expression contains all the literals or variables.
- In SOP each product term is called "minterm" and in POS each term is called "max term".
- The canonical form of SOP expression is called "minterm canonical" or "standard SOP" and the canonical form of POS is called as "max term canonical" or "standard POS"

eg: $Y = AB + \overline{A}B + \overline{A}\,\overline{B} + A\overline{B}$ (min term canonical)

$Y = (A+B)\cdot(\overline{A}+B)\cdot\cancel{(\overline{A}+\overline{B})}\cdot(A+\overline{B})$ (max term canonical)

$Y = \overline{A}\,\overline{B}\,\overline{C} + ABC + \overline{A}BC + \overline{A}\,\overline{B}C + AB\overline{C} + \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + A\overline{B}C$

Note: From the above example it is clear that for n variable total term should be 2^n

Q. Convert $X + X\overline{Y}$ into canonical form

$= X(Y+\overline{Y}) + X\overline{Y}$

$= XY + X\overline{Y} + X\overline{Y}$

$= XY + X\overline{Y}$

6. $X + Y\overline{Z}$ Convert to canonical form

$X(Y+\overline{Y})(Z+\overline{Z}) + YZ(X+\overline{X})$

$\Rightarrow (XY + X\overline{Y})(Z+\overline{Z}) + XYZ + \overline{X}YZ$

$\Rightarrow XYZ + X\overline{Y}Z + X\overline{Y}Z + XY\overline{Z} + XYZ + \overline{X}YZ$

$\Rightarrow XYZ + XY\overline{Z} + X\overline{Y}Z + \overline{X}YZ + X\overline{Y}\,\overline{Z}$

Conversion of POS in to Canonical Form

Rule 1: $A = (A+B)(A+\overline{B})$

Rule 2: $A+B = (A+B+C)(A+\overline{B}+\overline{C})$

Q. Convert $(A+B)(B+C)$ in to canonical form

$(A+B)(B+C)$

$\Rightarrow (A+B+C)(A+B+\overline{C})(A+B+C)(\overline{A}+B+C)$

Q. $X(Y+\overline{Z})$

$= (X+Y)(X+\overline{Y})(Y+\overline{Z})$

$= (X+Y+Z)(X+Y+\overline{Z})(X+\overline{Y}+Z)(X+\overline{Y}+\overline{Z})(X+Y+\overline{Z})(\overline{X}+Y+\overline{Z})$

Q. $X \cdot Y \cdot Z$

$= (X+Y)(X+\overline{Y})(X+Y)(\overline{X}+Y)(X+Z)(\overline{X}+Z)$

$= (X+Y+Z)(X+Y+\overline{Z})(X+\overline{Y}+Z)(X+\overline{Y}+\overline{Z})(X+Y+Z)$

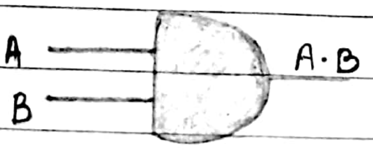
$(X+Y+\overline{Z})(\overline{X}+Y+Z)(\overline{X}+Y+\overline{Z})(X+Y+Z)(X+\overline{Y}+Z)$

$(\overline{X}+Y+Z)(\overline{X}+\overline{Y}+Z)$

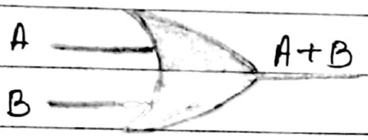
Logic Gates

- Logic gate is an electronic circuit that operates on one or more than one input signals to produce standard output
- Different logic gates are as follows

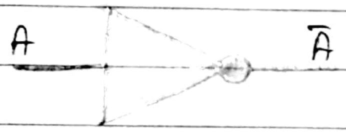
AND gate



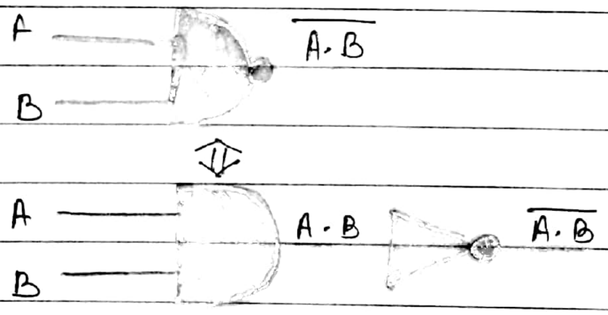
OR gate



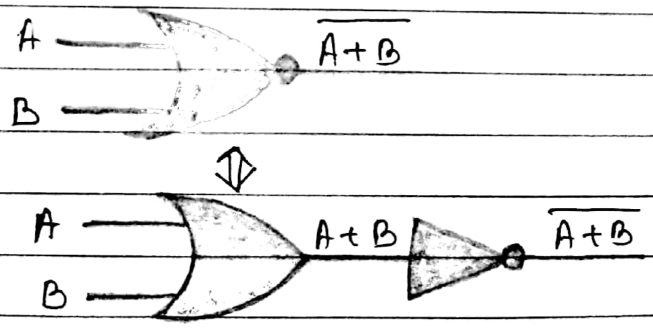
NOT gate



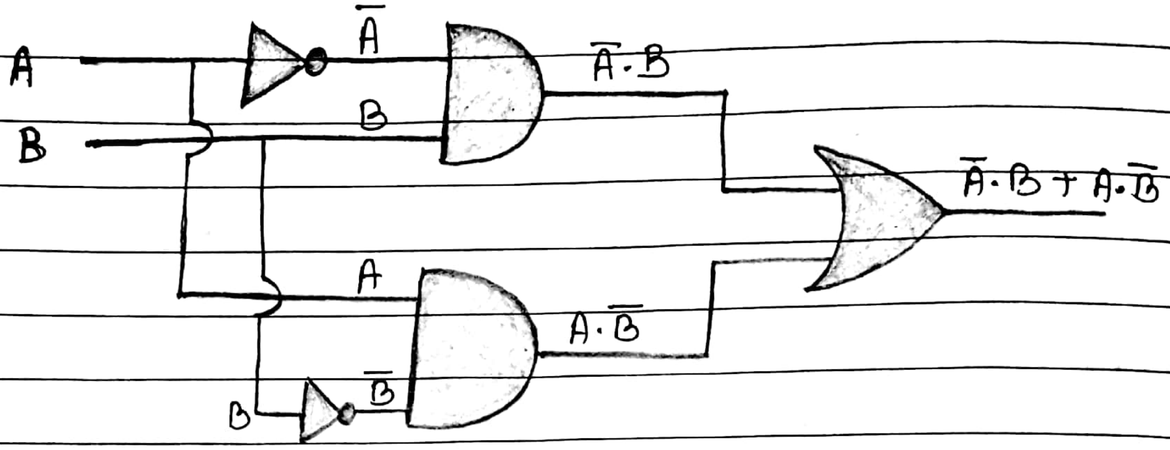
NAND gate



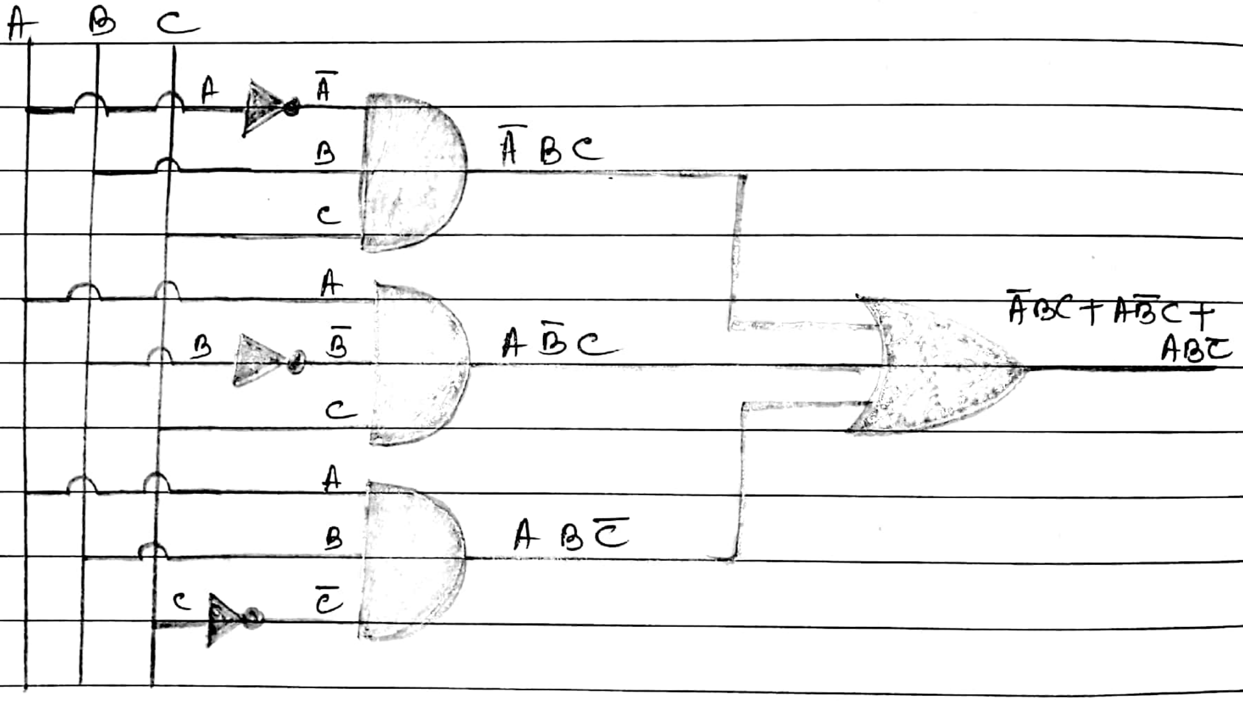
NOR gate



XOR gate $A \oplus B = \bar{A} \cdot B + A \cdot \bar{B}$



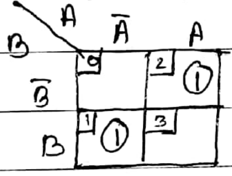
Q. $\bar{A}BC + A\bar{B}C + AB\bar{C}$



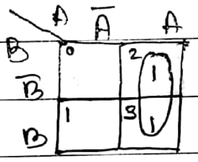
Minimisation of Boolean Expression by using K'Map

- It is a graphical method for simplifying boolean expression or method.
- It is a two dimensional representation of a truth table.
- It provides a simpler method for minimizing logic expressions or boolean expression.
- It concists of squares and each square represent a minterm or a maxterm.
- 'K' map for 'n' variables is made up of 2^n squares.

eg. K'Map of two variables



$B\bar{A} + \bar{B}A$



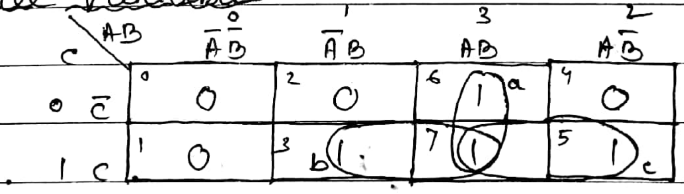
$AB + A\bar{B}$

$= A(B+\bar{B})$

$= A$

$Y = A$

K'map with three Variables



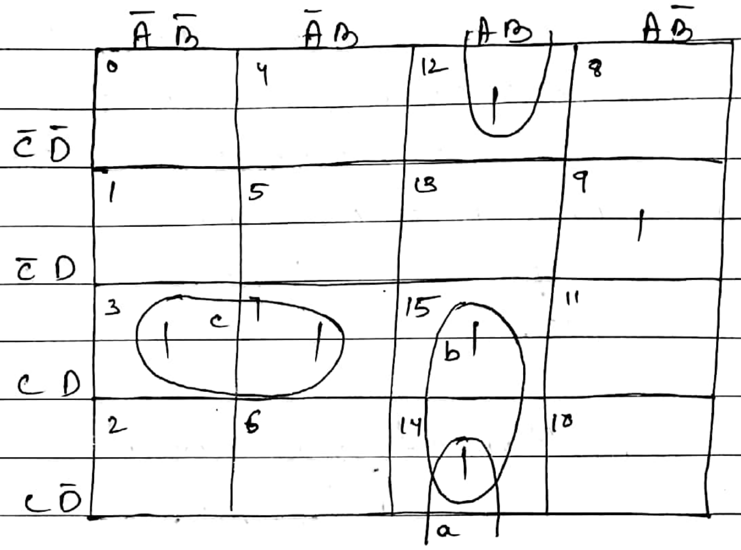
$Y = A\bar{B}C + ABC + \bar{A}BC + AB\bar{C}$

$a = AB$ $\quad b = BC$ $\quad c = AC$

$Y = AB + BC + AC$

K' Map with 4 variables

$Y = ABCD + A\bar{B}\bar{C}D + ABC\bar{D} + AB\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}BCD$



	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	0	4	12 1	8
$\bar{C}D$	1	5	13	9 1
CD	3 1	7 1	15 1	11
$C\bar{D}$	2	6	14 1	10

$a = AB\bar{D}$

$b = ABC$

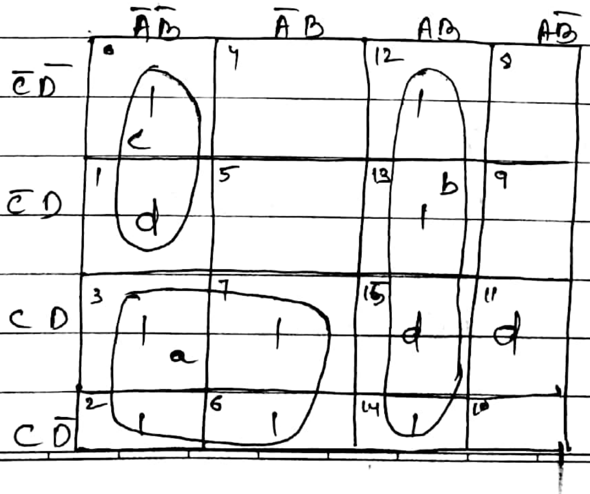
$c = \bar{A}CD$

$d = A\bar{B}\bar{C}D$

$Y = AB\bar{D} + ABC + \bar{A}CD + A\bar{B}\bar{C}D$

Q. Represent the following expression in k'map and mimize it.

$F(A, B, C, D) = \sum m(0, 2, 3, 6, 7, 12, 13, 14) + \sum d(1, 4, 11, 15)$



	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	0 1	4	12 1	8
$\bar{C}D$	1 d	5	13 1	9
CD	3 1	7 1	15 d	11 d
$C\bar{D}$	2 1	6 1	14 1	10

a = $\bar{A}C$

b = AB $\qquad$ $Y = \bar{A}C + AB + A\bar{B}C$

c = $A\bar{B}C$

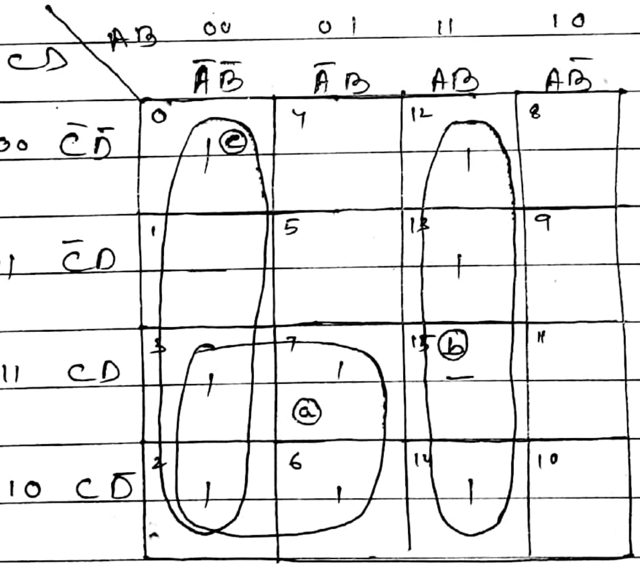
Q. What is "don't care" condition ?

ans: A dont care condition is indicated by putting a dash (—) or 'x' or 'd' in k'map and the ~~squares~~ squares with such entries are known as "don't care square".

- It will help for pairing with other pair at the time of simplification / pairing.

Q. Represent the boolean function / expression and find out minterms.

$F = (A, B, C, D) = \pi m(5, 8, 9, 10)\ \pi d(1, 4, 11, 15)$

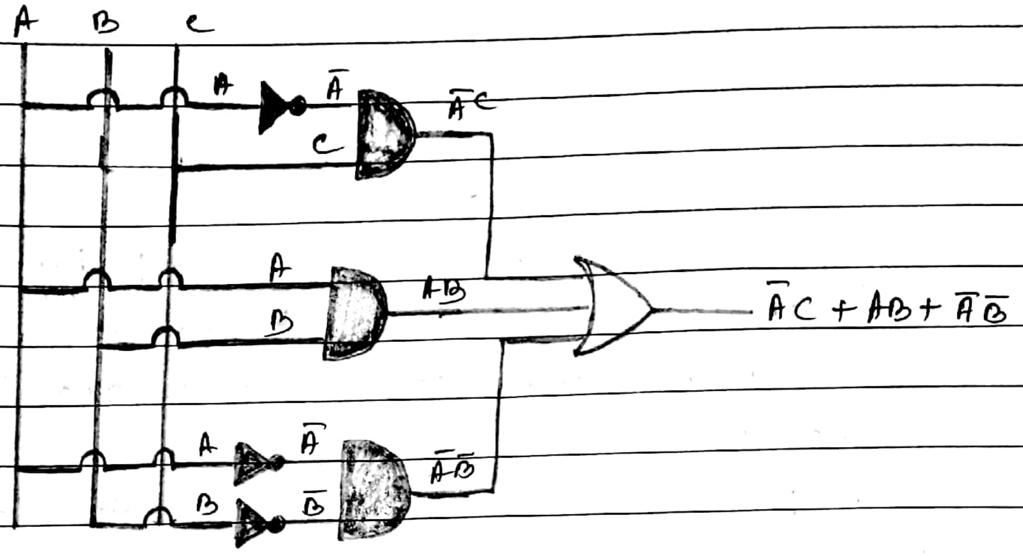


a = $\bar{A}C$

b = AB $\qquad$ $Y = \bar{A}C + AB + \bar{A}\bar{B}$

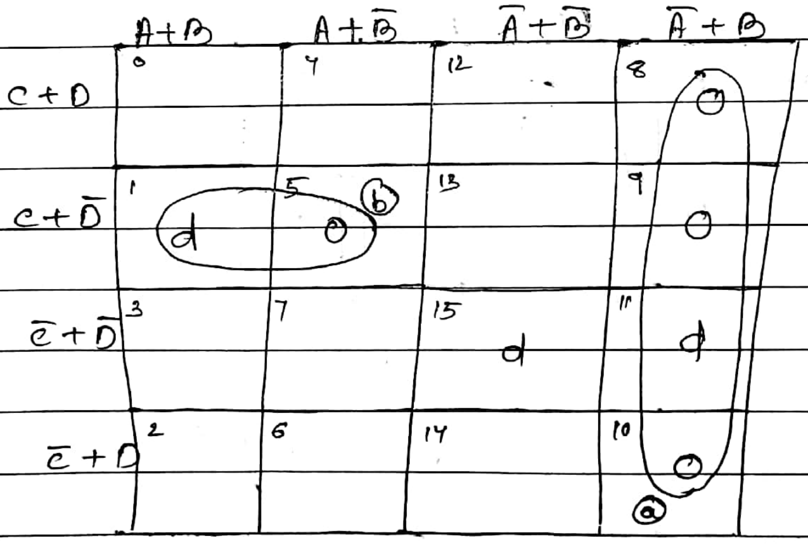
c = $\bar{A}\bar{B}$

$Y = \bar{A}C + AB + \bar{A}\bar{B}$



Q. Find out the maxterms of the following boolean expressions

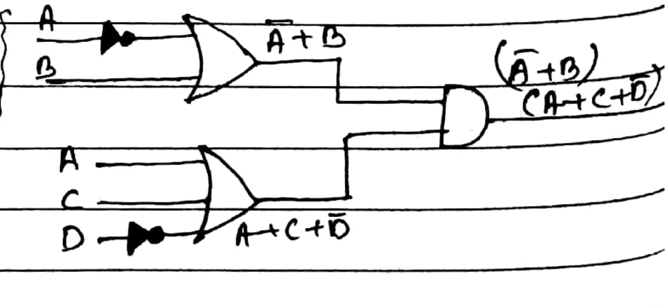
$F(A,B,C,D) = \pi_m(5,8,9,10) + \pi d(1,4,11,15)$



	$A+B$	$A+\bar{B}$	$\bar{A}+\bar{B}$	$\bar{A}+B$
$C+D$	0	4	12	8 0
$C+\bar{D}$	1 d	5 0	13	9 0
$\bar{C}+\bar{D}$	3	7	15 d	11 d
$\bar{C}+D$	2	6	14	10 0

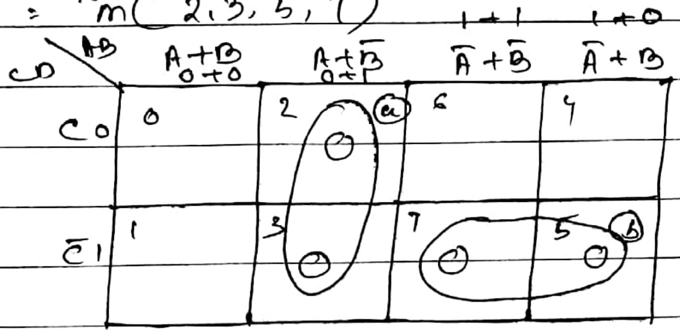
$a = \bar{A}+B$ $\quad$ $b = A+C+\bar{D}$

$Y = (\bar{A}+B)\cdot(A+C+\bar{D})$



Q. Represent the following boolean expression in k'map & solve it

$f(A,B,C) = \pi m(2,3,5,7)$



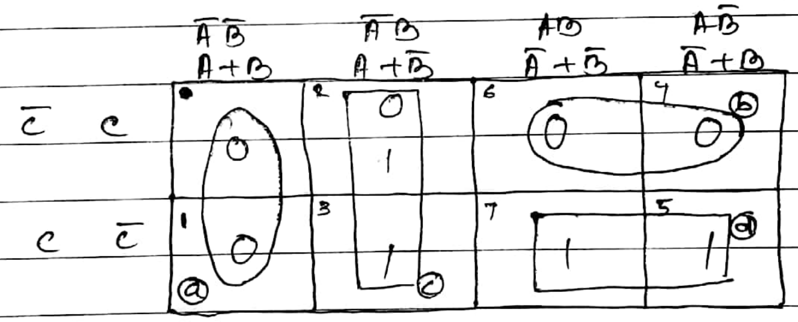
$a = A+\bar{B}$

$b = \bar{A}+\bar{C}$

$f = (A+\bar{B})(\bar{A}+\bar{C})$

Q. Findout the max term of the following boolean expression by using k-map.

$f(A,B,C) = \Sigma m(2,3,5,7)$



$a = A+B$ $\quad$ $c = \bar{A}B$

$b = \bar{A}+C$ $\quad$ $d = AC$

$f = (A+B)(\bar{A}+C)$ Maxterm $\quad$ $f = \bar{A}B+AC$ min term

GRAPH

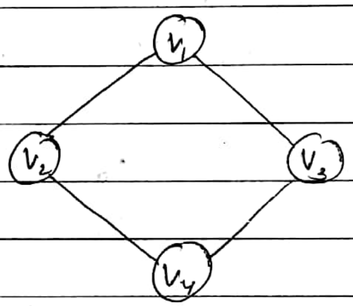
- It is a non-homogenous data structure

a graph can be represented as

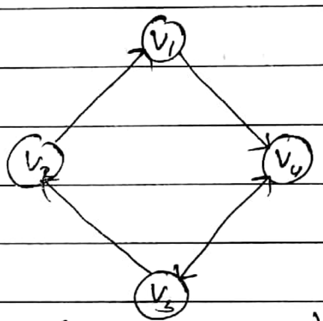
$$G = (V, E)$$

where G : the graph
V : set of the vertices $V = \{v_1, v_2, v_3 \dots v_n\}$
E : set of Edges
which contains the ordered pairs
i.e
$E = [(v_1, v_2)\ (v_2, v_4)\ (v_3, v_4) \dots]$

eg.



(undirected graph)



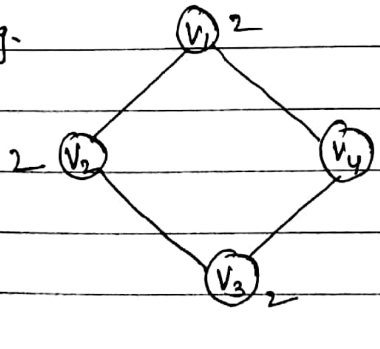
(directed graph)

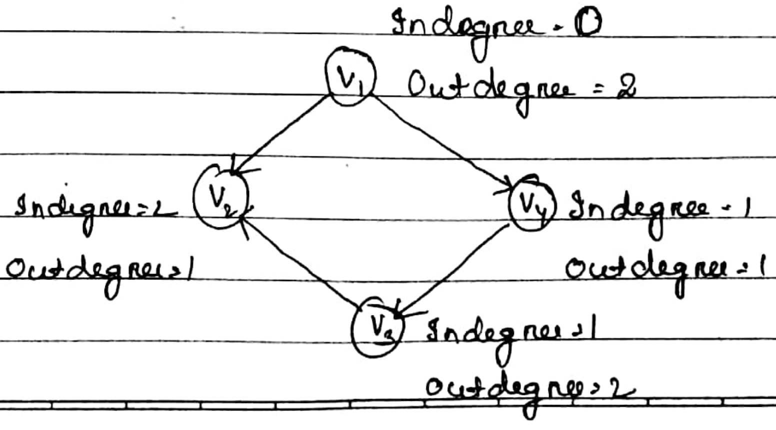
Graph Terminologies :

* Degree of Vertex :

- The degree of a vertex 'V' in a graph 'G' is the no. of edges incident with the vertex V.

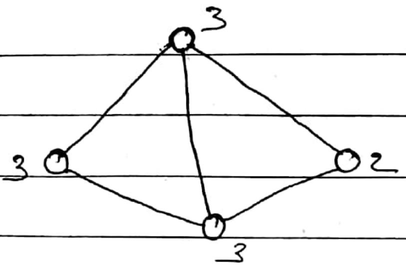
eg.





* Degree of Graph:

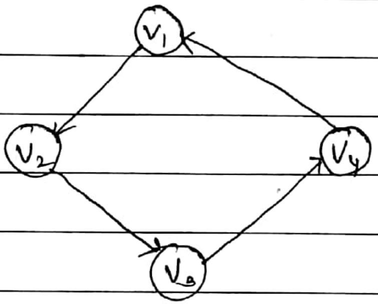
- It can be defined as the largest vertex degree of the graph



degree of graph = 3

* Adjecent Vertices

- A vertex V_i is said to be adjecent with the vertex V_j if there is a edge from V_i to V_j.

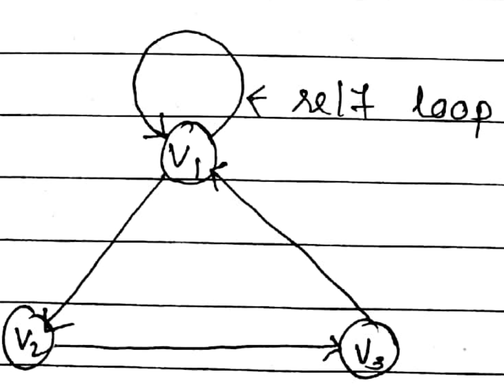


V_1 adjecent V_2
V_2 " V_3
V_3 " V_4
V_4 " V_1

* Self Loop.

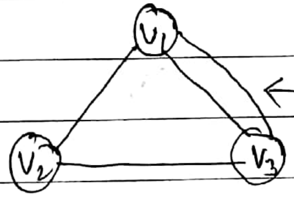
- If there is an edge ~~to~~ which starting and ending vertex is same then it is called self loop.

eg:



* **Parallel Edges:**

If there are more than one edges between two vertices / same pair of vertices then they are known as parallel edges or multiedges.

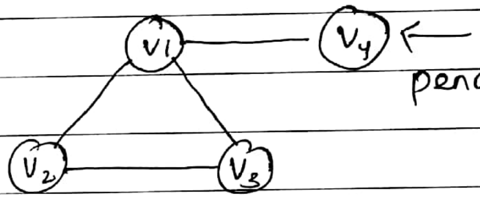


← parallel edges

* **Pendent Vertex:**

- If there is a single edge connected with the vertex then it is called a pendent vertex

(degree = 1)



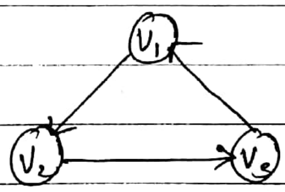
← pendent vertex

- The degree of pendent vertex is always 1.

Types of graph:

* **Directed Graph:** A graph G is said to be directed if the edge set is made of ordered vertex pair.

eg:



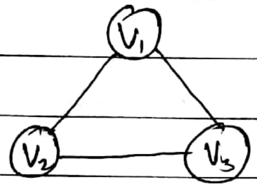
Here $G = \{V, E\}$ where $V = \{V_1, V_2, V_3\}$

$E = \{(V_1, V_2), (V_2, V_3), (V_3, V_1)\}$

* Undirected Graph:

- A graph G is said to be undirected if the edge set of unordered vertex pair.

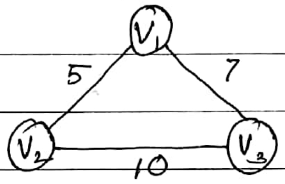
eg.



$E = \{(V_1, V_2), (V_2, V_1), (V_3, 1)\}$

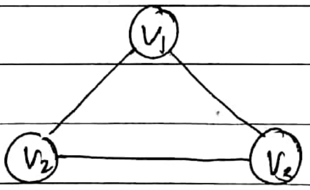
* Weighed Graph

- A graph is said to be weighted if all the edges are labelled or asigned with some weights



* Simple Graph

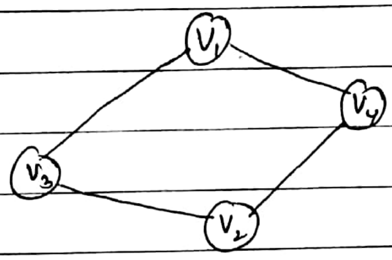
- A graph G is said to be simple if it does not contain any self loop or parallel edges



- The simple graph must be undirected.

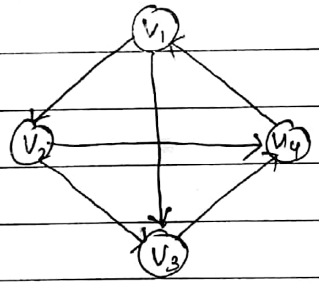
* Complete Graph

- A graph is said to be complete if each vertex v_i is adjecent to every other vertex v_j.



* Cyclic Graph

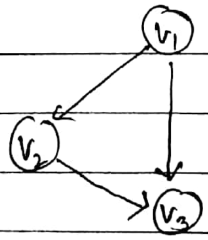
If there is a path containing one or more edges whose starting and end vertices are same then it is known as Cyclic Graph



- If a graph contain a cycle then it is called Cyclicgraph

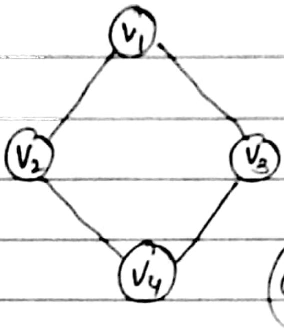
* Acyclic Graph

A graph having no cycle is called acyclic graph

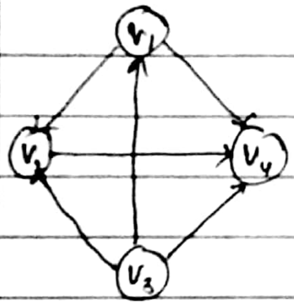


* Connected Graph :

- A graph is said to be connected if there is a path between any two vertices in a graph.



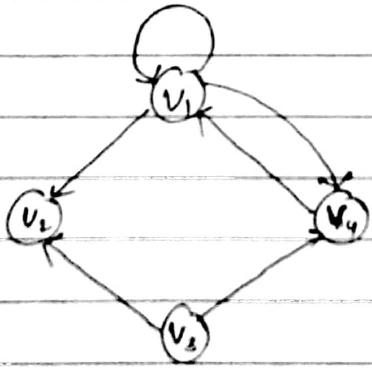
(connected / but not completed)



(not connected)

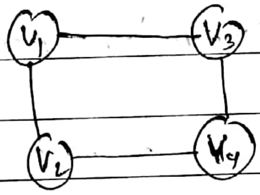
* Pseudograph

A graph contains self loop & parallel edges is called psuedo graph.

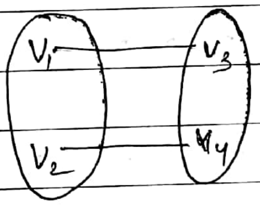


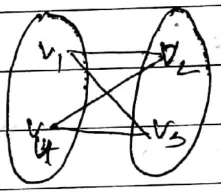
* Bipartite Graph

- A simple graph is said to be Bipartite if its vertex set V can be partitioned into two different sets V_1 & V_2 such that every edge in the graph connects a vertex V_1 & a vertex V_2 and if this condition holds then we call V_1, V_2 as a bipartition of the vertex set V of G.



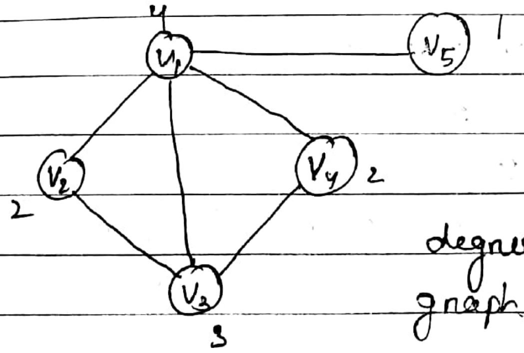
Ex





Degree Sequence of a Graph

- Let G be a graph with vertex set
$V(G) = V_1, V_2, V_3, \ldots V_n$
then the degree sequence of the graph can be defined as the sequence of degrees nos. in a non-increasing order i.e $\deg(V_1), \deg(V_2), \ldots \deg(V_n)$



degree sequence of this graph is 4 3 2 2 1

Graphical Sequence

- The sequence is said to be graphical, if it is a degree sequence of some graph.

Q/ Check the following degree sequence valid or not Check using Havel-Hakim theorem.

(i) 7 6 5 4 4 3 2 1
(ii) 6 6 6 6 3 3 2 2
(iii) 7 6 6 4 4 3 2 2

(i) ~~7~~ 6 5 4 4 3 2 1
↳ ~~5~~ 4 3 3 2 1 0
↳ ~~3~~ 2 2 1 0 0
↳ ~~1~~ 1 0 0 0
↳ 0 0 0 0

From the above it is clear that the degree sequence is valid.

(ii) ~~6~~ 6 6 6 3 3 2 2
↳ 5 5 5 2 2 1 2
↳ ~~5~~ 5 5 2 2 2 1
↳ ~~4~~ 4 1 1 1 1
↳ 3 0 0 0 1
↳ ~~3~~ 1 0 0 0
0 -1 -1 0 (negative not allowed)

the degree sequence is not valid

(iii) ~~7~~ 6 6 4 4 3 2 2

↳ ~~6~~ 5 3 3 2 1 1

↳ 4 2 2 1 0 1

~~4~~ 2 2 1 1 0

↳ * 1 0 0 0

↳ 0 0 0 0

From the above sequence it is clear that the degree sequence is valid.

Havel-Hakimi's Theorem :

- It is an algorithm in graph theory for solving graph realization problem.
- The algorithm was published by Havel in 1955 & later by Hakimi in 1962.

Stp-1 Let 'D' be a sequence $d_1, d_2, d_3 \cdots d_n$ with $d_1 \geq d_2 \geq d_3 \geq \cdots \geq d_n$

Stp-2 Let D' be the sequence obtained from D by

1) Disguarding d_1
2) Substract 1 from each of the next d_1 no. of entries of D'

Stp-3 Now D' is the sequence of $d_2 - 1, d_3 - 1, \cdots d_n - 1$

Stp-4 If D' is found as graphical then, continue the above process till the last in D' contain all zero otherwise return invalid

Stp-5 Exit

Page No.:

Date:

Representation of Graph

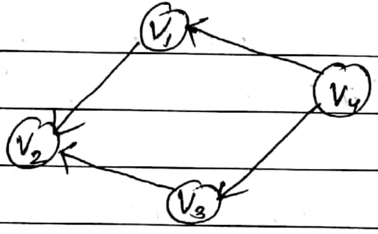
- A graph can be represented in memory in two ways
  i.e → Set Representation
  → Matrix Representation

→ **Set Representation :**

Let G be a graph and let $V = \{V_1, V_2, \cdots V_n\}$ be the set of vertices and
$E = \{(V_i, V_j)\}$ where $i, j = 1, 2, \cdots n$

then in set representation the graph can be represented as $G = \{V, E\}$

eg.



- In set representation the above graph can be represented as $G = \{V, E\}$
  where $V = \{V_1, V_2, V_3, V_4\}$
  $E = \{(V_1, V_2), (V_3, V_2), (V_4, V_3), (V_4, V_1)\}$

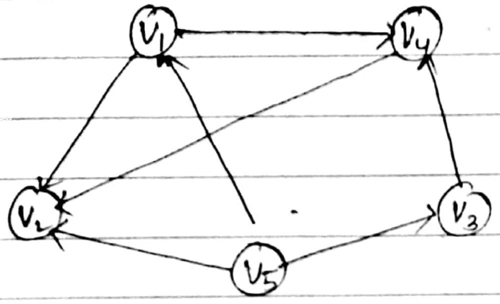
→ **Matrix Representation**

- Matrix representation in a graph can be classified in to three catagories.
  1. Adjacent Matrix
  2. Incident Matrix
  3. Path Matrix

1. Adjacent Matrix

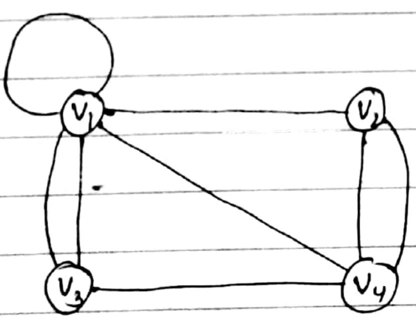
If "G" is a graph and if "a_{ij}" be the elements of the adjacent matrix of the graph "G" then it can be defined as

$$a_{ij} = \begin{cases} 1, & \text{if there is an edge from } i \text{ to } j \\ 0, & \text{otherwise} \end{cases}$$



the adjacent matrix of above graph can be represented as

$$\begin{array}{c|ccccc|} & V_1 & V_2 & V_3 & V_4 & V_5 \\ \hline V_1 & 0 & 1 & 0 & 1 & 0 \\ V_2 & 0 & 0 & 0 & 0 & 0 \\ V_3 & 0 & 0 & 0 & 1 & 0 \\ V_4 & 0 & 1 & 0 & 0 & 0 \\ V_5 & 1 & 1 & 1 & 0 & 0 \end{array}$$

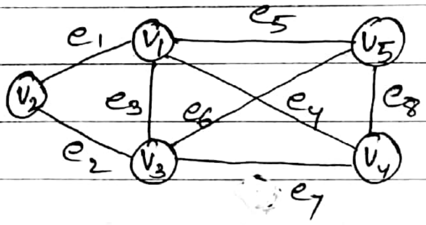


$$\begin{array}{c|cccc|} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 2 & 1 & 2 & 1 \\ V_2 & 1 & 0 & 0 & 2 \\ V_3 & 2 & 0 & 0 & 1 \\ V_4 & 1 & 2 & 1 & 0 \end{array}$$

2. Incident Graph

- If G is a graph and $e_1, e_2, e_3 \ldots e_n$ are the edges of the graph & if a_{ij} be the elements of the incident matrix of the graph then it can be defined as

$$a_{ij} \begin{cases} 1, \text{ if } e_j \text{ is incident on } v_i \\ 0, \text{ otherwise} \end{cases}$$

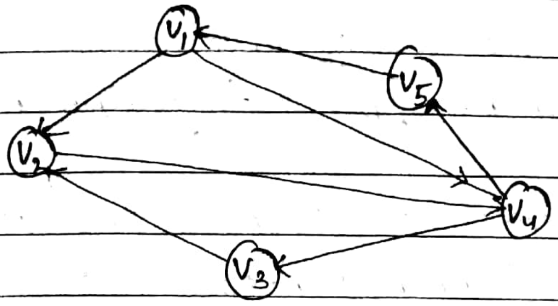


$$\begin{array}{c|cccccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \\ \hline V_1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ V_2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ V_3 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ V_4 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ V_5 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}$$

3. Path Matrix

- If "G" is a graph and if a_{ij} be the elements of the path matrix of the Graph "G" then it can be defined as

$$a_{ij} = \begin{cases} 1, \text{ if there is path from } i \text{ to } j \\ 0, \text{ otherwise} \end{cases}$$



$$\begin{array}{c|ccccc|} & V_1 & V_2 & V_3 & V_4 & V_5 \\ \hline V_1 & 1 & 1 & 1 & 1 & 1 \\ V_2 & 1 & 1 & 1 & 1 & 1 \\ V_3 & 1 & 1 & 1 & 1 & 1 \\ V_4 & 1 & 1 & 1 & 1 & 1 \\ V_5 & 1 & 1 & 1 & 1 & 1 \end{array}$$

Shortest Path Algorithms

- It is the procedure to find out the shortest path from initial node to goal node.
- In graphics it supports two algorithms

1. Dijkstar's Algorithms
2. Warshall's Algorithms

1. Dijkstar's Algorithms

Step by step procedure to solve a problem by using Dijkstra's Algorithm.

- It is a best technique to use to deatermine the shortest path between two arbitrary vertices in a graph.
- It is a single source shortest path algorithm. i.e it starts from any vertex as a source and finds out the minimum path to any other vertex in

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a graph alongwith the easist path
- This algorithm can be applied to graph having weights.

Stp-1 Select any vertex as source and asign it as 0 and asign all other vertex as ∞

Stp-2 Insert all the nodes in to an array named Q.

Stp-3 Select the value of the node that is minimum(V) From the Q and add it into the resulting set "S"

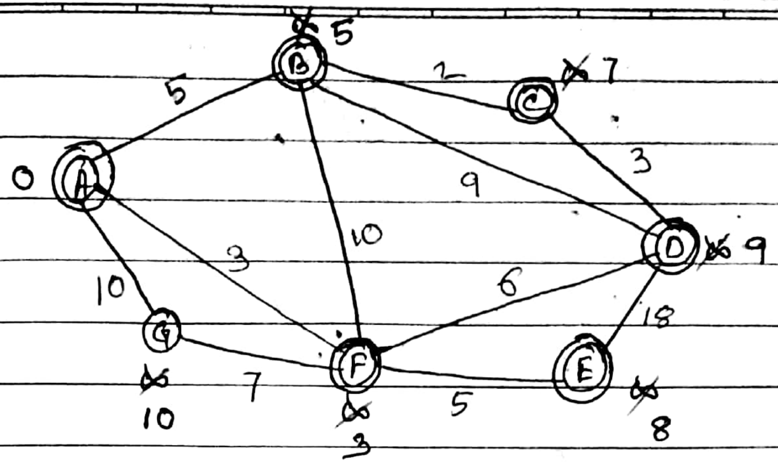
Stp-4 Continue the process untill the 'Q' is empty or all the nodes are inserted in to the set S.

Stp-5 Find out all the adjecent vertcies of "V" by using the Following Formula and update the distances of all the adjecent vertices by considering the minimum value

$$\text{if}\,(\text{dis}\,(V) > \text{dis}\,(u) + \text{length}\,(u.V))$$
$$\text{dis}\,(V) = \text{dis}\,(u) + \text{length}\,(u.V)$$

where V = the current node &
u = its adjecent nodes.

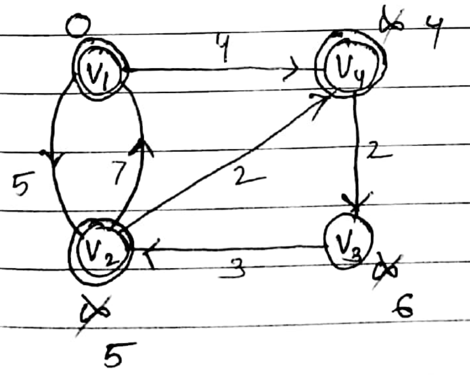
Stp-6 Now the set "S" contains all the vertices with minimum path distance From the source code eg:



S↓ Q→	A	B	C	D	E	F	G
A	$\boxed{0}$	∞	∞	∞	∞	∞	∞
B	-	5	∞	∞	∞	$\boxed{3}$	10
C	-	$\boxed{5}$	∞	9	8	-	10
D	-	-	$\boxed{7}$	9	8	-	10
E	-	-	-	9	$\boxed{8}$	-	10
F	-	-	-	$\boxed{9}$	-	-	10
G	-	-	-	-	-	-	$\boxed{10}$

{A B B C E D G}
0 3 5 7 8 9 10

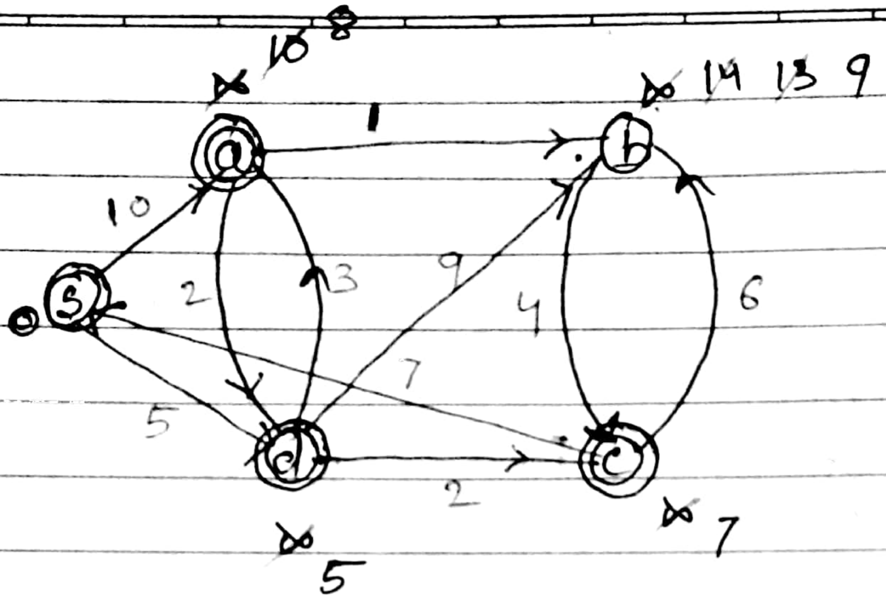
the final resulting set is S: { [illegible] }



S↓ Q→	V_1	V_2	V_3	V_4
V_1	$\boxed{0}$	∞	∞	∞
V_2	-	5	∞	$\boxed{4}$
V_3	-	$\boxed{5}$	6	-
V_4	-	-	$\boxed{6}$	-

0 4 5 6

the final resulting set is S: {V_1, V_4, V_2, V_3}



S→ G↓.	S	a	b	c	d
S	$\boxed{0}$	∞	∞	∞	∞
a	–	10	∞	∞	$\boxed{5}$
b	–	8	14	$\boxed{7}$	–
c	–	$\boxed{8}$	13	–	–
d	–	–	$\boxed{9}$	–	–

the final set S: $\{\overset{0}{s}, \overset{5}{d}, \overset{7}{c}, \overset{8}{a}, \overset{9}{b}\}$

Dijkstra Algorithm (G, W, S)

when G = the graph
S = the final resulting set and
W = all +ve weight

Stp-1 Initialise single source (G, S)
(It will intialise the source node i.e S)

Stp-2 S = ϕ
Initialisation of the resulting set matrix

Stp-3 Let Q = an array which contains all the vertex of the graph.

Stp-4
```
while Q ≠ 0
    {
        u = extract i.e min(Q)
        S = S ∪ {V}
    }
```

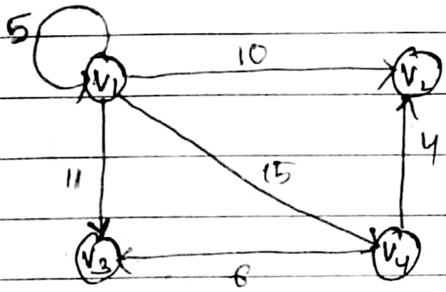
Stp-5 For each vertex V ∈ G , Adj(u) and relax (u, v, w)
```
relax (u, v, w)
{
    if (dis (V) > dis (u) + length(u, v)
    dis (V) = dis (u) + length (u, v)
    return
}
```

Weight Matrix

If G is a graph and a_{ij} be the elements of weight matrix of the graph then it can be difined as

$$a_{ij} = \begin{cases} a, & \text{if } a \text{ is a weight from } i \text{ to } j \\ 0. & \text{Other wise} \end{cases}$$

eg.



$$\begin{array}{c|cccc|} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 5 & 10 & 11 & 15 \\ V_2 & 0 & 0 & 0 & 0 \\ V_3 & 0 & 0 & 0 & 0 \\ V_4 & 0 & 4 & 6 & 0 \end{array}$$

Worshall's Algorithm.

Stp-1 Find out the weight matrix of the given graph and name as Q.

Stp-2 Substitute "∞" in place of "0" in Q and name as "Q_0"

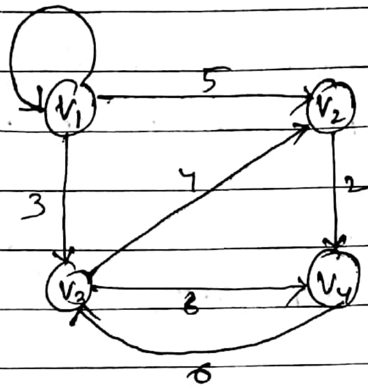
Stp-3 Find out all the path from the starting vertex (V_1) to all other vertex through V_1 and select the minimum and name the matrix as Q_1.

Stp-4 Find out all the paths from 2nd vertex (V_2) to all other vertexes and select minimum i.e either through V_2 or through V_1 & V_2

Stp-5 the above process will be continued untill all the nodes are selected or there will be infinity values in the matrix.

Stp-6 Exit

eg:



The weigh matrix of the above graph is

$$Q = \begin{array}{c|cccc|} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 7 & 5 & 3 & 0 \\ V_2 & 0 & 0 & 0 & 2 \\ V_3 & 0 & 4 & 0 & 8 \\ V_4 & 0 & 0 & 6 & 0 \end{array}$$

$$Q_2 = \begin{array}{c|cccc|} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 7 & 5 & 3 & \textcircled{7} \\ V_2 & \infty & \infty & \infty & 2 \\ V_3 & \infty & 4 & \infty & \textcircled{6} \\ V_4 & \infty & \infty & 6 & 6 \end{array}$$

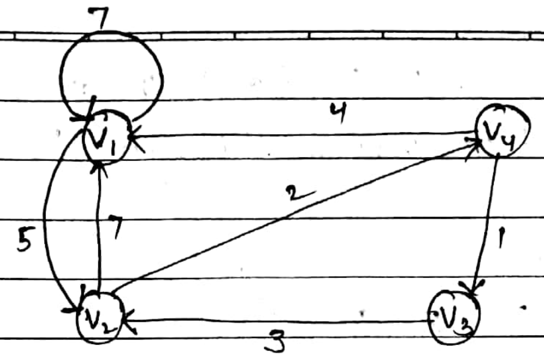
$$Q_0 : \begin{array}{c|cccc|} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 7 & 5 & 3 & \infty \\ V_2 & \infty & \infty & \infty & 2 \\ V_3 & \infty & 4 & \infty & 8 \\ V_4 & \infty & \infty & 6 & \infty \end{array}$$

$$Q_3 : \begin{array}{c|cccc|} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 7 & 5 & 3 & 7 \\ V_2 & \infty & \infty & \infty & 2 \\ V_3 & \infty & 4 & \infty & 6 \\ V_4 & \infty & \textcircled{10} & 6 & 12 \end{array}$$

$$Q_1 : \begin{array}{c|cccc|} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 7 & 5 & 3 & 8 \\ V_2 & \infty & \infty & \infty & 2 \\ V_3 & \infty & 4 & \infty & 8 \\ V_4 & \infty & \infty & 6 & \infty \end{array}$$

$$Q_4 : \begin{array}{c|cccc|} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 7 & 5 & 3 & 7 \\ V_2 & \infty & \textcircled{12} & \textcircled{8} & 2 \\ V_3 & \infty & 4 & \textcircled{14} & 6 \\ V_4 & \infty & 10 & 6 & 12 \end{array}$$

the above matrix represents the shortest path from one vertex to all other



Q. Find out the shortest path of the given graph from V_1 to V_4 using Worshall's Algorithm

Step-1 the weight matrix of the given graph

$$G = \begin{array}{c|cccc|} & V_1 & V_2 & V_3 & V_4 \\ V_1 & 7 & 5 & 0 & 0 \\ V_2 & 7 & 0 & 0 & 2 \\ V_3 & 0 & 3 & 0 & 0 \\ V_4 & 4 & 0 & 1 & 0 \end{array}$$

Step-2 Substitute ∞ in place of zero

$$G_0 = \begin{array}{c|cccc|} & V_1 & V_2 & V_3 & V_4 \\ V_1 & 7 & 5 & \infty & \infty \\ V_2 & 7 & \infty & \infty & 2 \\ V_3 & \infty & 3 & \infty & \infty \\ V_4 & 4 & \infty & 1 & \infty \end{array}$$

Step-3 Now we will find out the minimum path from V_1 to all other vertex through V_1

$$\begin{array}{c|cccc|} & V_1 & V_2 & V_3 & V_4 \\ V_1 & 7 & 5 & \infty & \infty \\ V_2 & 7 & \infty & \infty & 2 \\ V_3 & \infty & 3 & \infty & \infty \\ V_4 & 4 & \infty & 1 & \infty \end{array}$$

Stp-4 Now we will findout the minimum path from V_1 to all other vertex through V_2 or V_1, V_2.

$$A_2 = \begin{array}{c|cccc} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 7 & 5 & \infty & ⑦ \\ V_2 & 7 & ⑫ & \infty & 2 \\ V_3 & 10 & 3 & \infty & ⑤ \\ V_4 & 4 & ⑨ & 1 & ⑪ \end{array}$$

Stp-5

$$A_3 = \begin{array}{c|cccc} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 7 & 5 & \infty & 7 \\ V_2 & 7 & 12 & \infty & 2 \\ V_3 & 10 & 3 & \infty & 5 \\ V_4 & 4 & ⑨ & 1 & ⑥ \end{array}$$

Stp-6

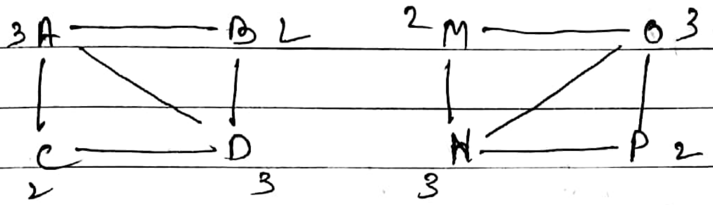
$$\begin{array}{c|cccc} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 7 & 5 & ⑧ & 7 \\ V_2 & ⑥ & ⑥ & ③ & 2 \\ V_3 & ⑨ & 3 & ⑥ & 5 \\ V_4 & 4 & 4 & 1 & 6 \end{array}$$

the final resulting matrix is

$$\begin{array}{c|cccc} & V_1 & V_1 & V_3 & V_4 \\ \hline V_1 & 7 & 5 & 8 & 7 \\ V_2 & 6 & 6 & 3 & 2 \\ V_3 & 9 & 3 & 6 & 5 \\ V_4 & 4 & 4 & 1 & 6 \end{array}$$

* Isomorphic Graph

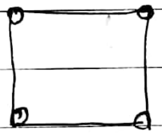
- A graph is said to be isomorphic if they have (i) same vertices and ~~eg~~ edges ② degree variance i.e the degree present in both the graphs must be same.



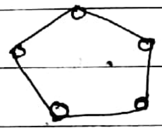
* Hamiltonian Path & Circuit

- A simple path in a graph G, that passes through every vertex or edge exactly once is called Hamiltonian Path. A circuit in Graph G that passes through every vertex exactly once is called Hamiltonian Circuit.

Hamiltonian Path

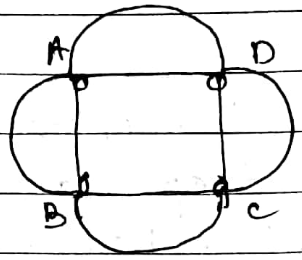


Hamiltonian Circuit



* Eular's Circuit & Path

- An Eular's Circuit on a graph G is a simple circuit containing every vertex of G and an Eular Path is a simple path in a graph containing every edge of G
- The Connected Multigraph with atleast two vertices has an Eular circuit if and only if its vertices has even degree.

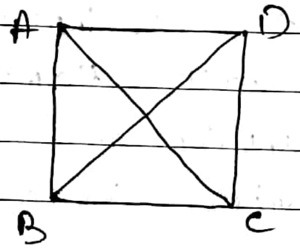


(Eulan Cincuit & Graph)

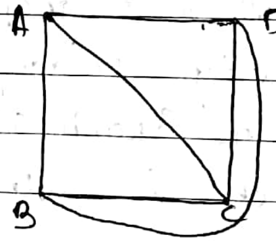
Plannen Graph

A graph is called plannen graph if it can be drawn with any edges not crossing eachother.

Let us take an example of k_4 graph

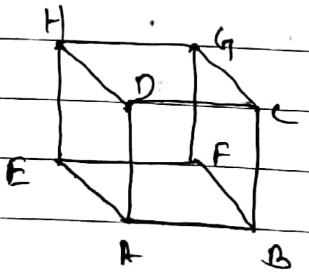


(Graph)

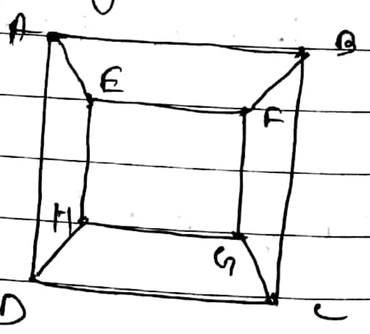


(plannen Graph)

Let us take an example of Q_3 graph

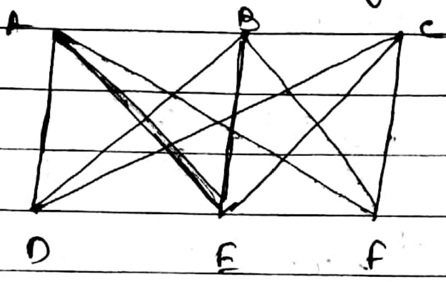


(Q_3 graph)



(Plannen graph)

Let us take another $K_{3,3}$ graph



→ Not a planner graph

⊛ planarity of a graph plays an important role in design of electronics circuit.

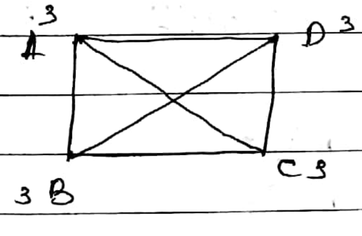
Handshaking Theorem :

- Let us consider a graph "G" which is undirected and $G = (V,E)$ where V is the vertices and E is the set of edges then Handshaking theorem can be defined as " the sum of degrees of the vertices is twice the no of edges".

Mathematically it can be defined as

$$\boxed{2 \cdot E = \sum_{\in V} deg(\,)}$$

eg!



total no of degree = 12
total no of edges = 2×6 = 12

Q. Show that an undirected graph has an even no. of vertices of odd degree

proof! Let us consider the two vertex sets V_1 & V_2 such that V_1 contains all the vertices even degrees and V_2 contains all the vertices of odd degrees.

i.e $\sum_{\in V_1} deg(\,)$ = Even degrees —— ①

$\sum_{\in V_2} deg(\,)$ = odd degrees —— ②

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According to Handshaking theorem we can define the above problem as

$$2 \cdot E = \sum_{\in V_1} deg(\,) + \sum_{\in V_2} deg(\,) \text{ --- } ③$$

In equn ③ the 1^{st} part contain even no. of vertices and the left hand side part must be even no. So by considering the above conclusion the 2nd expression of right hand ~~right~~ side part should be even in equn ③

this concluded that an undirected graph has an even no. of vertices of odd degree

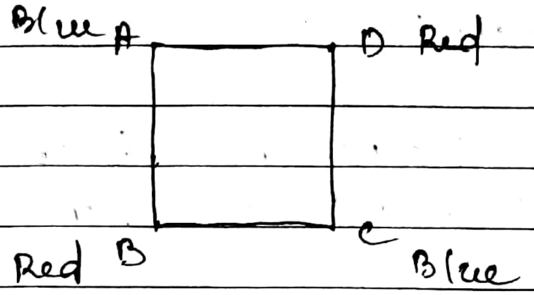
Graph Colouring

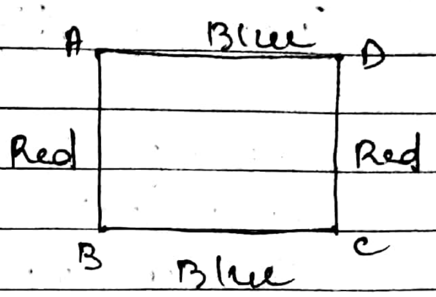
- A colouring of a simple graph is the asignment of colour to each vertex or edges of a graph so that no two adjecent vertices or edges are asigned the same colour.
- Graph colouring can be classified in to two categories.

→ Vertex Colouring
→ Edge Colouring

Vertex Colouring : a graph should not contain any two adjecent vertex as the same colour.

Edge colouring : a graph should not contain any two adjecent edges as the same colour.

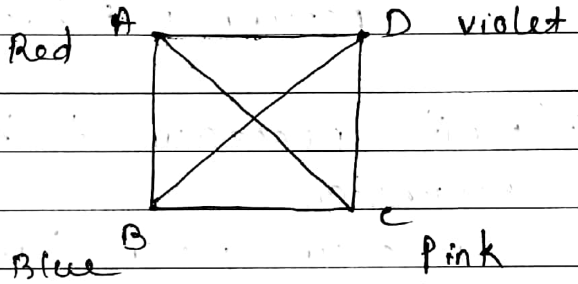




(vertex colouring) (Edge colouring)

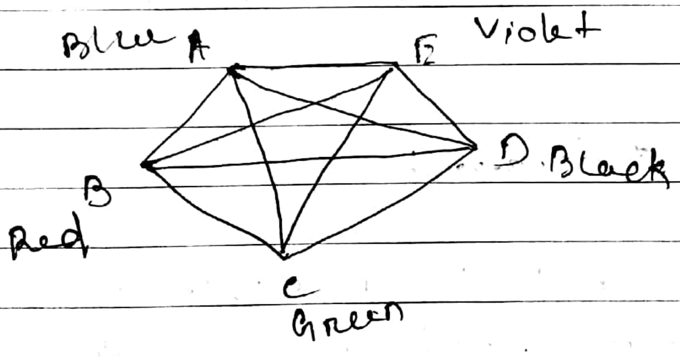
Chromatic Number of a graph is the least no of colours needed for colouring the graph and it is denoted as by by special character
χ ( ) (chi)

Q. Find out the chromatic no of K_4 graph



$\chi(K_4) = 4$
$\chi(G) = 4$

Q. Find out the chromatic no. of K_5 graph



Recursive Algorithm:

- An algorithm is called recursive if its solves a problem by reducing it to an instance of same problem with smaller input.

Example: to write a algorithm by recursion to find out factorials of a given number.

```
Procedure factorial (n)
        if (n = 0)
        return 1;
        else
        return n * factorial (n-1)
```

Q/ write procedure to find out a^n by using recursion.

```
Procedure power (a, n)
if (n = 0)
 return 1
 else
 return a * power (a, n-1)
```

Structural Induction:

- We use mathematical over the set of +ve integers and recursion definition to prove the result about a recursively defined sets.
- How ever instead of using mathematical induction directly to prove the results about recursively defined sets then you can use a more convinient form of induction called structural induction.

Q/ Give a recursive definition of the sequence $\{a_n\}$ where $n = 1, 2, 3 \ldots\ldots$, & $a_n = 4n-2$.

$a_n = 4n - 2$

at $n = 1$, $a_1 = 4.1 - 2 = 2$

$n = 2$, $a_2 = 4.2 - 2 = 6$

$n = 3$, $a_3 = 4.3 - 2 = 10$

$\boxed{a_{n+1} = a_n + 4}$

by asuming the above recursive step we can find out $a_{n+1} = a_n + 4$ for all n.

Sequences :

-Sequences are ordered list of elements.
A sequence is a discrete structure use to represent an order list and it may be a finite sequence or may be an infinite sequence.

1, 2, 3, 4, 5, 6 (finite sequence of 6 elements)
1, 2, 3, 4, 5, 6 (infinite sequence)

-A sequence can be denoted as $\{a_n\}$ to represent a sequence
ex: the sequence $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots\ldots$
can be represented as $a_n = 1/n$

sequences can be categorised into two types
- Geometric progression
- Arithmatic progression

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- A Geometric Progression is a sequence in the form of $a, ar, ar^2, ar^3 \ldots$ where a = initial term
r = real nos

- An Arethmatic Progression is a sequence in the form of $a, a+d, a+2d, a+3d, \ldots$
where a = initia term
d = common difference & a real no.

Summations:

- In summations we can describe the notations used to express the sum of the terms used in a sequence.
ex- suppose a sequence $a_m, a_{m+1}, \ldots a_n$ can be represented in summation as

$$\sum_{i=m}^{n} a_n$$

where i = index of summation
m = Lower limit
n = upper limit

Q/ express the the sum of 1st hundred terms of the sequence
when $a_n = \frac{1}{n}$ and $n = 1, 2, 3 \ldots$

$$\sum_{n=1}^{100} \frac{1}{n} = 1/1 + 1/2 + 1/3 + 1/4 + \cdots + 1/100$$

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Groups and Rings:

Let A be a set and ⊕ be any binary operation then (A,⊕) is said to be a group if it satisfies the following properties

1. **Closure:**
   This property tells if $(a,b) \in A$ then $(a \oplus b) \in A$

2. **Associative:**
   This property tells if $(a,b,c) \in A$, then $(a \oplus b) \oplus c = a \oplus (b \oplus c)$

3. **Identity:**
   This property tells if $a \in A$, then there exist some elements 'e' such that $a \oplus e = a$

4. **Inverse:**
   This property tells that if $a \in A$ then there exist some element "a^{-1}" such that $a \oplus a^{-1} = e$

5. **Commutative:**
   This property tells that $(a,b) \in A$ then $a \oplus b = b \oplus a$

EX- Show that $G = \{0,1,2,3,4,5\}$ & '$*$' be a additin modul
6 is a group.

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

Closure : From the above table it is clear that all the elements belongs to G and hence satisfied i.e $(a,b) \in G$ then a addition modulo 6 $b \in G$

associative : From the above table it is clear that it satisfies the property of associative i.e $(a*b)*c = a*(b*c)$

Let $a=2$, $b=4$, $c=5$

LHS $(2*4)*5$

$= 0 * 5$ It will return modulo 6 of $(2*4)$

$= 5$

RHS $2*(\overset{4}{\cancel{b}}*\overset{5}{\cancel{c}})$

$= 2 * 3$ It will return modulo 6 of $(4*5)$

$= 5$

LHS = RHS

identity : From the table it is clear that the 1st row and the 1st column elements are same and hence it satisfies properties of identity i.e $a \in G$ and $e \in G$ then $a*e = a$

inverse : From the above table it is clear that it satisfies the properties of inverse i.e $a \in G$ and $a^{-1} \in G$ then $a * a^{-1} = e$

$0 * 0 = 0$
$1 * 5 = 0$
$2 * 4 = 0$
$3 * 3 = 0$
$4 * 2 = 0$
$1 * 5 = 0$

Commutative(x): From the above table it is clear that it satisfies the properties of commutati i.e $a, b \in G$ then $a * b = b * a$

Let $a = 2$, $b = 5$

LHS $a * b = 2 * 5 = 1$

RHS $b * a = 5 * 2 = 1$

As it satisfies all the properties of a group so G is a group.

* **Abelian Group:**

Let A be any set and '$*$' be any binary operation then $(A, *)$ is said to be abelian group if it satisfies the properties of closure, identity, inverse, associative as well as commutative.

* **Groupoids:** Let 'A' be any set and '$*$' be any binary operation then $(A, *)$ is said to be groupoid if it satisfies only closure property.

* **Monoid:** Let 'A' be any set and '$*$' be any binary operation then $(A, *)$ is said to be monoid if it satisfies the properties of closure, associative and identity.

* **Semigroup:** Let 'A' be any set and '$*$' be any binary operation then $(A, *)$ is said to be semigroup if it satisfies the properties of closure and associative.

* **Group:** Let 'A' be any set and '$*$' be any binary operation then $(A, *)$ is said to be group if it satisfies closure, identity associative and inverse properties.

closure } groupoid/ Algebric structure } semigroup } monoid } group
associative
identity
Inverse
commutative

Abeliangroup

$G = \{0,1,2,3,4,5\}$ addition modulo 6

$G = \{0,1,2,3,4,5\}$, $+_6$, $(G,*)$ or $(G, +_6)$

Q/ check the following expression for group /monoid/ semigroup /algebric structure

$A = \{0,1,2,3,4\}$ $\times_5$

ans:

closure :

From the table it is clear that $\forall\ a,b \in A$ then $a \times_5 b \in A$ ⇒ it satisfies closure property

$\times_5$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	3	1	2
3	0	3	2	4	2
4	0	4	3	2	1

associative :

as we know multiplication satisfies associative so it is clear that it will

$\forall\ a,b,c \in A$ then $a \times_5 (b \times_5 c) = (a \times_5 b) \times c$

Let $a=2,\ b=3,\ c=4$

$2 \times_5 (3 \times_5 4) = (2 \times_5 3) \times_5 4$

$LHS = 2 \times_5 (3 \times_5 4)$ $RHS = (2 \times_5 3) \times_5 4$

$= 2 \times_5 2$ $= 1 \times_5 4$

$= 0$ $= 0$

$LHS = RHS$

Hence, it satisfies the property of associative.

identity:

this property defines if $a \in A$ and $e \in A$ then $a * e = a$

Here let us consider $e = 1 \in A$ and it is clear that all element identity property.

$0 \times_5 1 = 0$

$1 \times_5 1 = 1$

$2 \times_5 1 = 2$

inverse:

As '0' is present in the set as an element then it will never satisfies the inverse property.

From the above it is concluded that the set is a Monoid.

Q/(i) $\{0, 1, 2, 3, 4\}, +_5$ (Abelian Group)

(ii) $\{1, 2, 3\}, \times_5$ (closure not satisfied)

(iii) $\{1, 2, 3\}, +_5$

(iv) $\{0, 1, 2, 3, 4, 5\}, +_6$

(v) $\{0, 2, 4\}, +_6$ (Group)

(vi) $\{1, 3, 5, 7\}, +_8$

(vii) $\{1, 3, 5, 7\}, \times_8$

Q/ Let Q^+ = set of all +ve Rational nos and $(Q^+, *)$ is an abelian group and $a * b = \frac{a \cdot b}{3}$ then check the followings as true or false.

(i) $e = 3$ (ii) $a^{-1} = 9/a$ (iii) $(2/3)^{-1} = 6$ (iv) $3^{-1} = 3$

(i) $a * e = a$

$\Rightarrow \frac{a \cdot e}{3} = a$

$\Rightarrow e = 3$ (true)

(ii) $a^{-1} = 9/a$

$a * a^{-1} = e$

LHS $\frac{a \cdot a^{-1}}{3}$

$= \frac{a \cdot 9/a}{3} = 3$

$= e =$ RHS

(iii) $(2/3)^{-1} = 6$

$\Rightarrow (2/3)^{-1} \cdot 6 = 3$

$\Rightarrow 3/2 \cdot 6 = 3$

$\Rightarrow 9 \neq 3$

Q/ Let N is a natural no. and $(a*b) = a^b$ the check $(N, *)$ is a group / semigroup

ans:

Closure: it defines $\forall a, b \in A$, then $a*b \in A$

Here $\forall a, b \in N$ then $a^b \in N$

i.e let $a=2$; $b=3$

$a*b = 2^3 = 8 \in N$

associative: it defines $\forall a, b, c \in A$, then

$(a*b)*c = a*(b*c)$

so here $(a*b)*c = a*(b*c)$

LHS $(a*b)*c$ RHS $= a*(b*c)$

$= (a^b)*c$ $= a*b^c$

$= a^{b^c}$ $= a^{b^c}$

LHS = RHS

it satisfies associative property

identity: it defines if $a \in A$, $e \in A$ then

here $a*e = A$

$a*e = a^e \neq a$

$\Rightarrow$ it does not satisfy identity property

so it is a semigroup

Q/ If z is a set of integers and $a*b = max(a, b)$ then show that $(z, *)$ is a group.

closure:

It defines $a, b \in z$ then $a*b \in z$

Here $(a*b) = max(a, b)$ if a and b are integer the maximum of the integer must be an integer and which is true

Ex: let $a = 3$, $b = 5 \in z$

$3*5 = max(3, 5) = 5 \in z$

it satisfies the property of closure

associative:

it defines a $a, b, c \in z$ then $(a * b) * c = a * (b * c)$

LHS $= (a * b) * c$

$= \max(a * b) * c$ $\qquad a < b < c$

$= b * c$

$= \max(b, c)$

$= c$

RHS $= a * (b * c)$

$= a * (\max(b, c))$

$= a * c$

$= \max(a, c)$

$= c$

LHS $=$ RHS

it satisfies the property of associativity

identity:

it defines if $a \in z$ and $e \in z$ then $a * e = a$

here $a * e = \max(a * e)$

$= \max(a, -\infty)$

$= a$ (which is not allowed)

it doesnot satisfy identity

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Ring :

A set R is said to be a ring if (R, *) must be an Abelian Group

(i) (R, +) must be an abelian group

(ii) (R, ·) must be a semigroup

(iii) it must satisfy the distributive law

$$a \cdot (b+c) = a \cdot b + a \cdot c$$

$$(a+b) \cdot c = a \cdot c + b \cdot c$$

The ring represented as (R, +, ·)

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Binomeal Coefficient :-

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$$

$$= \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y^1 + \binom{n}{2}x^{n-2}y^2 + \cdots\cdots$$

$$+ \binom{n}{n-1}x \cdot y^{n-1} + \binom{n}{n}y^n$$

Here x and y be 2 variables, let 'n' be a non-negative integer then the Binomeal theorem can be defined as the above.

Q/ What is the expansion of $(x+y)^4$
as we know from the binomeal theorem

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j \text{ —— } ①$$

$$(x+y)^4 = \sum_{j=0}^{4} \binom{4}{j} x^{4-j} y^j$$

$$= \binom{4}{0}x^4y^0 + \binom{4}{1}x^3y^1 + \binom{4}{2}x^2y^2 + \binom{4}{3}x^1 \cdot y^3$$

$$+ \binom{4}{4}x^0 \cdot y^4$$

$$= \frac{\cancel{4!}}{0!\,\cancel{4!}}x^4 + \frac{4!}{1!\,3!}x^3y^1 + \frac{4!}{2!\,2!}x^2y^2 + \frac{4!}{3!\,1!}x \cdot y^3 +$$

$$\frac{\cancel{4!}}{\cancel{4!}\,0!}1 \cdot y^4$$

$$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

Q/ What is the co.efficient of $x^{12}y^{13}$ in the expansion of $(x+y)^{25}$

$$(x+y)^n = \sum_{j=0}^{n}\binom{n}{j}x^{n-j}y^{j} \text{ ——— } ①$$

if $j = 13$, $n = 25$ then coefficient of $x^{12}y^{13}$ is $\binom{25}{13}$ i.e $\dfrac{25!}{13!\ 12!}$

$$= \frac{25\times24\times\cdots14\times13\times\cancel{12!}}{13!\times\cancel{12!}}$$

$$= \frac{25\times24\times\cdots14\times\cancel{13}}{\cancel{13}\times12!}$$

$$= \frac{25\times24\times\cdots\times14}{12!}$$

Q/

Q/ What is coefficient of $x^{12}y^{13}$ in the expansion of $(2x-3y)^{25}$?

ans: As we know from the binomean theorem.

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j \quad \text{— ①}$$

$$\text{i.e } (2x+(-3y))^{25} = \sum_{j=0}^{n} \binom{n}{j} 2x^{n-j} (-3y)^j$$

$$\text{at } x^{12} y^{13} = \binom{25}{13} (2x)^{12} \cdot (-3y)^{13} \quad \text{— ②}$$

$$\text{the co-efficient is } \binom{25}{13} 2^{12} \cdot (-3)^{13}$$

$$= \frac{25!}{13!\,12!} (2)^{12} (-3)^{13}$$

Q/ Let n be a non-negative integer

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n$$

$$2^n = (1+1)^n$$
$$= \sum_{j=0}^{n} \binom{n}{k} 1^{n-k} 1^n$$
$$= \sum_{k=0}^{n} \binom{n}{k} 1 \cdot 1$$
$$= \sum_{k=0}^{n} \binom{n}{k} = LHS$$

Q/ Let n be a +ve integer then show that

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$$

RHS. $0 = (1+(-1))^n$ $\left\{ \because (x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k \right\}$

$\Rightarrow \sum_{k=0}^{n} \binom{n}{k} (1)^{n-k} (-1)^k$

$= \sum_{k=0}^{n} (-1)^k \binom{n}{k} = \text{LHS}$

Q/ Let n is a non-negative integer then show that

$$\sum_{k=0}^{n} 2^k \binom{n}{k} = 3^n$$

RHS. $3^n = (1+2)^n$ $\left\{ \because (x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k \right\}$

$= \sum_{k=0}^{n} \binom{n}{k} (1)^{n-k} \cdot (2)^k$

$= \sum_{k=0}^{n} 2^k \binom{n}{k} = \text{LHS}$

H/W. 1. Find the co-efficients of $x^5 y^8$ in $(x+y)^{13}$.

2. Find the co-efficient of $x^8 y^9$ in $(3x+2y)^{17}$.

3. Find the expansion of $(x+y)^5$.

1. as we know from the binomeal theorem

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^{i}$$

if $j=8$ and $n=13$, equ^n becomes

$$\binom{13}{8} x^5 y^{13}$$

$\therefore$ Coefficient of $x^5 y^8 = \binom{13}{8}$

$$\binom{13}{8} = \frac{13!}{5!\,8!} = \frac{\cancel{8!}\times \overset{3}{\cancel{9}}\times \overset{2}{\cancel{10}}\times 11\times \overset{3}{\cancel{12}}\times 13}{\cancel{8!}\times \cancel{5}\times 4\times \cancel{3}\times \cancel{2}\times 1}$$

$= 1287$ (ans)

2. as we know from the binomeal theorem

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^{j} \quad — ①$$

$$\Rightarrow (3x+2y)^{17} = \sum_{j=0}^{17} \binom{17}{j} (3x)^{17-j} (2y)^{j} \quad — ②$$

if $j=9$, $n=17$, then, equ^n ② becomes

$$\binom{17}{9} (3x)^8 (2y)^9$$

coefficient of $x^8 y^9$ is

$$\binom{17}{9} 3^8\, 2^9 = \frac{17!}{9! \times 8!} \times 3^8 \times 2^9$$

3. as we know from the binomeal theorem

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$$

$$(x+y)^5 = \sum_{j=0}^{5} \binom{n}{j} x^{5-j} y^j$$

$$= \binom{5}{0} x^5 + \binom{5}{1} x^4 y + \binom{5}{2} x^3 y^2 +$$

$$\binom{5}{3} x^2 y^3 + \binom{5}{4} x y^4 + \binom{5}{5} y^5$$

$$= \frac{5!}{0!\,5!} x^5 + \frac{5!}{1!\,4!} x^4 y + \frac{5!}{2!\,3!} x^3 y^2 + \frac{5!}{3!\,2!} x^2 y^3 +$$

$$\frac{5!}{4!\,1!} x y^4 + \frac{5!}{5!\,0!} y^5$$

$$= 5x^4 y + 10x^3 y^2 + 10x^2 y^3 + 5x y^4 + y^5$$

Inclusion & Exclusion Principle

- This is otherwise called as substraction principle i.e suppose a task can be done in N_1 or N_2 ways but some of the ways to do the N_1 work is same as N_2 work.
- In this situation we can not use the sam rule to count the number of ways to do the task and to solve the situation we can apply a technique called inclusion-exclusion principle.
- This principle can be defined as $|N_1 \cup N_2| = |N_1| + |N_2| - |N_1 \cap N_2|$

Q/ A computer company recived 350 applications from computer graduates for a job planning. Suppose 220 of these people are specialised in Comp. Sc, 147 are specialised in business and 51 are specialised in both. So how many how many of these application neither majored in computer science nor in business.

ans:

total application recived = 350

A_1 = specialised in computerscience = 220

A_2 = " " Business = 147

By using Inclusion-Exclusion principle

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$
$$= 220 + 147 - 51$$
$$= 316$$

Teacher's Signature ....

$\overset{34}{20}$ neither received degree in Computerscience nor in business.

Permutation and Combination

Q/ Suppose, there are ~~[illegible]~~ 8 nos in a race and the winner receives a gold medal, the 2nd winner get silver and the 3rd one a bronze. Then how many ways the awards can be distributed among the player.

ans:

Here 3 medals can be distributed in different ways among 8 people/players. So here we can apply the method of permutation. i.e $P(8,3) = \frac{8!}{5!} = 8 \times 7 \times 6 = 336$

$$\left\{ P(n,r) = \frac{n!}{(n-r)!} \right\}$$ (Formula for permutation)

Q/ How many permutations of the letter ABCDEFGHI contains string AB ?

ans: An AB indicates a single character so total no. of permutations from the 7 objects can be done in 7! ways i.e $7! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040$ ways.

Q/ A group of people have been trained to perform a specific task. How many ways to select a group of ~~[illegible]~~ six expert people to go ~~th~~ to the mission.

ans: To solve the above problem we can use the method of combination i.e $C(30,6) = \frac{30!}{6!\ 24!} = \frac{30\times29\times28\times27\times26\times25}{6\times5\times4\times3\times2\times1}$

Q/ Let n and r be two non-negative integers with $r \leq n$ then show that $C(r,n) = C(n,n-r)$

LHS. $C(n,r) = C(n,n-r)$ —— ①

RHS. $C(n,n-r) = \frac{n!}{(n-r)!\,(n-(n-r))!}$

$$= \frac{n!}{(n-r)!\ r!} =$$

$=$ LHS.

Conditional Probability :-

- If E and F are two events associated with the sample space in the random experiment then the conditional probability of even E, given F has already occured then it is denoted as

$$\boxed{P(E/F) = \frac{P(E\cap F)}{P(F)}}$$ ✓

Q) Find out the probability of getting exactly one head by tossing a coin twice.

ans:-

To toss a coin twice we can get the following sample space $S = \{HH, HT, TH, TT\}$

Let E = an event exactly one head the

$E = \{HT, TH\}$

F = an event with atleast one head

$= \{HH, HT, TH\}$

Now $P(E) = \frac{1}{2}\times\frac{1}{2} + \frac{1}{2}\times\frac{1}{2}$

$= \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2}$

$P(F) = \frac{1}{2}\times\frac{1}{2} + \frac{1}{2}\times\frac{1}{2} + \frac{1}{2}\times\frac{1}{2}$

$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

$= \frac{3}{4}$

By applying conditional probability $P(E/F) = \frac{P(E\cap E)}{P(F)} = \frac{P(E)}{P(F)}$

$= \frac{1/2}{3/4} = 2/3$ ans

Q/ A family has two children what is the probability that both the children are boys, given that at least one of them is boy.

ans:

The sample space for the above problem is

$S = \{bb, bg, gb, g g\}$

Let $E =$ Event with both are boys $= \{bb\}$

$F =$ event with atleast one boy

$= \{bb, bg, gb\}$

$P(E) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(F) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$

Now, applying conditional probability

$$P(E/F) = \frac{P(E)}{P(F)} = \frac{1/4}{3/4} = \frac{1}{3} \quad \text{(an}$$

Baye's Theorem :-

- $E_1, E_2, \ldots E_n$ are n, non-empty events and these events constitute a partition of sample space 'S' i.e $E_1, E_2 \ldots E_n$ are pairwise disjoint and $E_1 \cup E_2 \cup \ldots \cup E_n$ and A is any event with non-zero probability i.e

$$\boxed{P(E_i/A) = \frac{P(E_i)\,P(A/E_i)}{\sum_{j=1}^{n} P(E_j)\,P(A/E_i)}}$$

Proof:
From the conditional probability we know that

$$P(A/E_i) = \frac{P(A \cap E_i)}{P(E_i)} \Rightarrow P(A \cap E_i) = P(A/E_i) \cdot P(E_i) \quad \text{①}$$

Similarly

$$P(E_i/A) = \frac{P(E_i \cap A)}{P(A)}$$

$$\Rightarrow ~~P(E_i\cap A)~~ \; P(E_i/A) = \frac{P(E_i \cap A)}{P(A)}$$

$$\Rightarrow P(E_i \cap A) = P(E_i/A) \cdot P(A)$$

$$\Rightarrow P(A \cap E_i) = P(E_i/A) \cdot P(A) \quad \text{②}$$

From the above derivation it is clear that

$$P(E_i/A) \cdot P(A) = P(A/E_i) \cdot P(E_i)$$

$$\Rightarrow P(E_i/A) = \frac{P(A/E_i) \cdot P(E_i)}{P(A)} \quad \text{③}$$

As we know from the principle of mutual Exclusion

$$P(A) = \sum_{j=1}^{n} P(E_j) \cdot P(A/E_j) \quad \text{④}$$

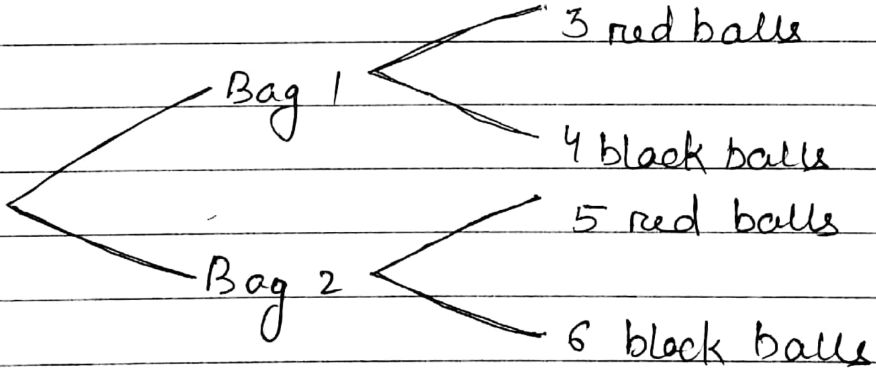
on substituting the value of equn④ in equ③

$$P(E_i/A) = \frac{P(A/E_i)\cdot P(E_i)}{\sum_{j=1}^{n} P(E_j)\cdot P(A/E_i)}$$

Q/ Bag 1 contain 3 red and 4 black balls while another bag contains 5 red and 6 black balls, 1 ball is drawn at random from one of the bags and it is found to be red, find probability that the ball has drawn from bag 2.

Sol^n :-

Here we can impleament the concept of ~~base~~ Baye's theorem so before applying the theorem over 1st task is draw the tree diagram of the problem

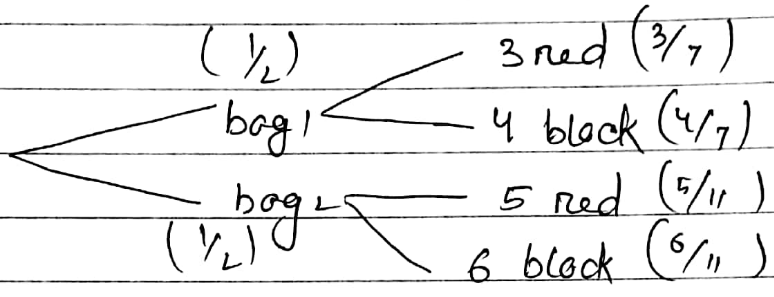


Let A : the drawn ball is red

E_1 : bag one is choosen

E_2 : bag two is choosen

From the above ~~Al~~ tree diagram it is clear that the different probabilities of getting red and black balls is



Now acording to baye's theorem

$$P(E_2/A) = \frac{P(A/E_2)\cdot P(E_2)}{\sum_{j=1}^{2} P(E_j)\cdot P(A/E_j)}$$

$$P(E_2/A) = \frac{P(A/E_2)\cdot P(E_2)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$$

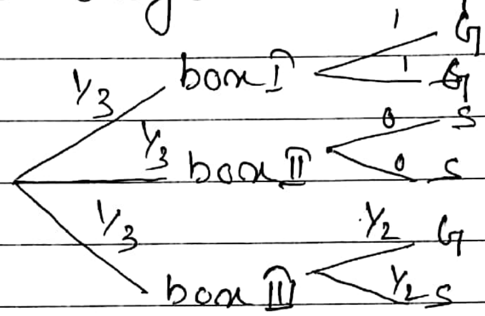
$$= \frac{1/2 \times 5/11}{1/2 \times 3/7 + 1/2 \times 5/11} = \frac{5/22}{3/14 + 5/22} = \frac{5/22}{68/154} = \frac{35}{154}$$

Q/ Given that three identical boxes 1,2,3 and each box has two coins, in box 1 both the coins are gold, in box 2 both the coins are silver and in box 3 one coin is gold and another is silver.

A person chooses a box at random and takes out a coin. If a coin is gold, then what is probability that other coin in the box is also a gold.

Soln:

the tree diagram of the problem can be done as



Let A : choosen coin as gold

E_1 : box 1 selected

E_2 : box 2 selected

E_3 : box 3 selected

the tree diagram for getting a gold from any of the box is written above.

by applying Baye's algorithm

$$P(E_1/A) = \frac{P(E_1) . P(A/E_1)}{\sum_{j=1}^{3} P(E_j) . P(A/E_j)}$$

$$= \frac{P(E_1) . P(A/E_1)}{P(E_1) . P(A/E_1) + P(E_2) . P(A/E_2) + P(E_3) P(A/E_3)} = \frac{1/3 . 1}{1/3 \times 1 + \frac{1}{2} \times 0 + 1/3 \times \frac{1}{2}}$$

$$= \frac{1/3}{1/3 + 1/6} = 2/3$$

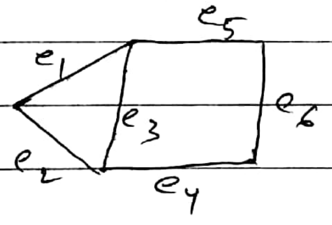
Teacher's Signature ....

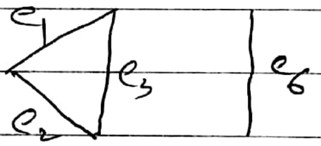
Discrete Probability

- A function that asign a number to each outcome is called a random variable and the random variables are the result of experiments, observations and surveys.
- A random variable that has a finite no. of outcomes is called discrete random variables and the probability that describes the probability of the ~~out comes~~ occurances of each discrete random variable is called discrete probability.

UNIT. 4

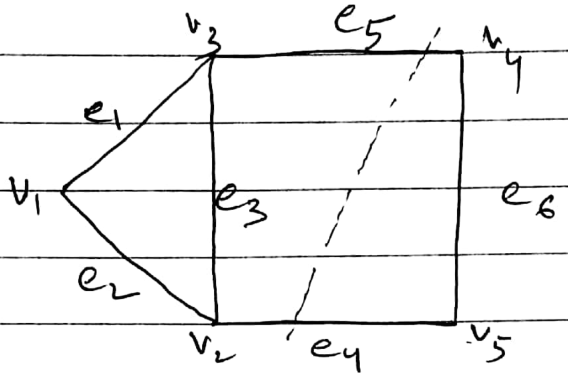
Cutset of Graph:





$E'_1 = \{e_5, e_4\}$

Let $G = (V, E)$ be a graph where $V = \{V_1, V_2, V_3 \ldots V_n\}$ and $E = \{e_1, e_2, \ldots e_n\}$ then the Cutset of the Graph can be defined as a set of edges "E'" and $E' \subset E$ which devides the graph in to two sub parts i.e after deleting the cutsets or set of edges the graph becomes disconnected



$V = \{V_1, V_2, V_3, V_4, V_5\}$

$E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$

E_1' = cutset = $\{e_5, e_4\}$